



Examining the Effects of Using Polya's Problem-solving Model on Mathematical Academic Achievement and Analyzing Ability of the Fourth Grade Students

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Authors' contributions

This work was carried out in collaboration between both the authors. Author UP designed the study, performed the statistical analysis, literature searches, wrote the protocol and wrote the first draft of the manuscript. Author YD managed data collection and analysis for the study. Both authors read and approved the final manuscript.

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ABSTRACT

The study sought to investigate the effects of Polya's Problem-Solving Model (PPSM) on learning achievement and analyzing ability in mathematics of fourth grade students. The sample of 32 students out of three sections was selected purposively as the subject of the study. Pretest and posttest one-group experimental design, achievement test and time series record were used to collect data. The collected data were analyzed using descriptive statistic and t-test. The t-test analysis revealed a statistical significant difference between the mean scores of in favor of posttest ($M=13.78$; $SD=3.02$) than the pretest ($M=6.66$; $SD=2.44$) at $p=.00$, indicating the improvement in the mathematical academic achievement using PPSM. Moreover, the time series record also showed that there was significant progression in the performances of the students after the inclusion of the PPSM. Thus, this study recommends the implementation of PPSM as an alternative method of instruction in teaching and learning of mathematical problem solving.

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1. INTRODUCTION

The primary goal of mathematics teaching and learning is to develop the ability to solve variety of complex problems that students encountered in real life. In normal classroom teaching, however, the common practice found in mathematics learning of students was that they were very much used to doing exercises without full understanding of the procedures involved in thinking and analyzing process. As a result, when students encountered a real problem, they were not able to apply the thinking and analyzing skills to solve the problems. In mathematics, understanding and transferring of the knowledge and skills to a new situation play a vital role. The poor performance of the students in mathematics problem solving was the primary indicator of inappropriate learning or lack of problem solving skills. This also indicated that the teaching procedures or methodologies adopted by the teachers were not able to help improve their problem solving skills. Students need to be trained using effective problem solving procedures and analyzing ability. Also, the students should be further encouraged to come up with their own creative strategies by moving beyond what the teacher teaches. Above all, the general standard of mathematics problem solving states that students should; a) build a new mathematical knowledge through problem solving, b) solve problem that arises in mathematics and other context, c) apply and adopt a variety of appropriate strategies to solve problem and, d) monitor and reflect on the process of mathematical problem solving.

The past studies using PPSM in various subjects found the model very effective (e.g., Perveen [1]; Ali [2]; Eksi [3]; Bowman [4]; Huang et al. [5]. The model consists of four steps 1. Understand the problem: students must know what is known, what is unknown and the ultimate goal 2. Devise a plan: students must understand how what they know connects to what they attempt to determine. They should understand precisely how to proceed. 3. Carry out: they should perform the mechanics of solving. 4. Look back: students must be sure to examine their solution and learn from what they have accomplished. Thus, this study was carried out to find the effectiveness of PPSM on learning achievement and analyzing ability in mathematics of the fourth grade students. Moreover, there is limited

literature in the Bhutanese context that have applied those models in enhancing solving mathematic word problem. Because of the need for research in this area, this study was conducted to evaluate effectiveness of PPSM as the instructional method in teaching mathematics problem solving.

1.1 Objectives of the Study

- i. To find out academic achievement in mathematics after using the Polya's Problem-Solving Model.
- ii. To find out the analyzing ability in mathematics using PPSM.

1.2 Research Questions

- I. Does Polya's Problem-Solving Model have effect on Mathematical academic achievement of the fourth grade students?
- II. Does Polya's Problem-Solving Model have effect on the analytical ability of the fourth grade students?

2. POLYA'S PROBLEM-SOLVING MODEL

Amongst many models, George Polya's Problem-Solving Model is one which is widely used. In his renowned publication, "How to Solve It", Polya [6] suggested that solving a problem involved: i) Understanding the problem; ii) Devising a plan; iii) Carrying out the plan; iv) Looking back. He described the problem solving process as a linear progression from one phase to the next and advocated that when solving a problem;

First, students have to see clearly what is required. They have to identify known and unknown information provided in the problem. According to Polya [7], student should not only understand the problem but also desire its solution. He believes that teachers can check the understanding to some extent by asking questions.

Second, students have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution and to make a plan. Polya mentioned that there are many possible ways to solve a problem. The skill at choosing the appropriate strategy is by solving many problems. The materials necessary for solving mathematics

problem are certain relevant items of formally acquired knowledge, formerly solved problem and formerly proven theorem.

Third, students carry out the plan. The plan provides the outline however the plans need to be thoroughly examined one after another, patiently. Persistent with the plan that student has chosen, if it continues not to work student can discard it and choose another.

Fourth, students have to look back at the completed solution; we review it and discuss it. The problem is often not completely understood until the problem-solver has tried and failed to arrive at a solution using different strategies Polya [6]. He mentioned that much can be gained by taking time to reflect and look back at what you have done, what worked and what didn't. It is a series of going forward and backward among the four stages of the model. This will help to predict what strategies to use to solve future problem. The figure below represents Polya's Problem-Solving Model.

2.1 Related Research

Perveen [1] conducted a study in the Government Girls High School Rawalpindi to find the Effect of the PPSM approach on academic achievement of students in mathematics at the secondary level. The sample size consisted of 48 students. The experimental group was taught using a series of lesson plans put together with the help of Polya [7] guidelines, which include heuristic steps of the problem-solving approach. The result of the finding revealed the effectiveness of the problem-solving approach.

Bowman [4] also conducted a research on evaluating word problem using Polya's Problem-Solving strategies. His study was attempted to determine whether Polya's Problem-Solving plan was an effective strategy for improving middle school ESL (English as second language) students' oral and written communication of their mathematical thoughts when solving word problems. The study demonstrates that Polya's plan helps ESL (English as a second language) students organize their thinking before trying to convey their message and the study also noted that the teaching of Polya's problem-solving strategy improved ESL students' abilities to understand what information is important in a math word problem and what information students still need to find. Ali [2] examined the

effect of PPSM on students' achievement in teaching mathematics at secondary level. 76 eight grade students of Government Girls Higher Secondary School Ghoriwala, Bannu, Pakistan, were taken as the sample of the study. Pretest and posttest design was used to collect data and it was analyzed using mean, standard deviation and t-test. The finding of the study concluded that there was significant difference between the traditional teaching method and problem solving method. It also revealed that problem solving method enhanced the achievement of students' mathematics.

Eksi [3] carried out study to investigate the effect of different problem-solving strategies on freshman university students' achievements of quantitative problems in a general chemistry course. In order to identify students' achievements of quantitative problems in chemistry, the Quantitative Problem-solving Achievement Test consisting of 20 multiple-choice items was developed and administered to 150 students as pretest and posttests. The results of ANOVA indicated that the students' achievements of quantitative problems in the Polya's Problem-Solving strategy incorporating with cooperative learning approach were better than the students' achievement of quantitative problems in traditional problem-solving and the Polya's problem-solving strategies. Based on the findings, it can be concluded that this approach helps the development of students' problem-solving skills and achievements because using the Polya's problem-solving strategy with cooperative learning may increase verbal interaction among students.

Schurter [8] conducted study on "Comprehensive Monitoring and Polya's heuristics as tools for problem solving". His study comprises of three groups and was taught using different methodologies. The first group was taught using traditional method (controlled), second group was taught using comprehensive monitoring alone (experimental) and the third group was taught with conjunction of Polya's 4-step method (experimental). He used pretest, posttest, interview and questionnaires to collect the information. The information gathered was analyzed using t-test and ANOVA. The result of the finding stated that students who received increase emphasis on the use of comprehensive monitoring alone and in conjunction with Polya's heuristics both seems to perform better in mathematic problem-solving then those who did not receive either type of instruction.

Uche [9] conducted a study to investigate the effect of problem solving instructional method on the mathematics achievement. In his research he used pretest and posttest to collect data. The data was analyzed using SPSS. The study found out that PPSM was more effective and beneficial to the students.

The study carried out by Gray [10] to investigate the impact of the application of Polya's first two steps to problem-solving have on an advanced high school mathematics classroom. The study employed mixed method action research design with 20 high school students for eight consecutive weeks. The 20 students in the class showed an increase in their mean scores from pretest to posttest. The same twenty students showed an increase in the number of strategies used on each problem from pretest to posttest.

Newfoundland [11] conducted a study of the effect of teaching heuristics on the ability of 10th grade students to solve novel mathematical problems. A group of ten boys were taught by the use of self-instruction booklets to apply the heuristics of examination of cases and analogy to novel mathematical problems. The data, analyzed by ANOVA, indicates that: 1) students can be taught to apply at least one heuristic to a novel problem; 2) it is better to teach heuristic alone than to combine the instruction with the teaching of mathematical content; and 3) the ability to apply at least one heuristic is independent of the vehicle used to introduce it. The evidence suggests that heuristic-oriented instruction can be an effective mode for teaching mathematical problem-solving.

3. METHODOLOGY AND RESEARCH INSTRUMENTS

The purposive sampling was done and one section of 32 students was taken as the subject of the study. This one group of experimental was taught using the PPSM as the strategy for five consecutive weeks. Pretest, posttest and time series records were the instruments used to collect the data. The achievement test was used for assessing students' performance which would

reflect their level of knowledge and analyzing ability in mathematics before and after the treatment. During the treatment, class activities were carried out mostly in groups (mixed ability). For the time series record, a test consisting of two questions was conducted at the end of every lesson. It was assessed using Polya's Problem-Solving rubric. Each question carried 16 points (4 points for each step). At first, the students faced problem to follow the Polya's model as they were used to solving problem using the old method where they skipped the first two steps of Polya's Model. But with repeated practice and reminder they could follow all the steps. The validity of test items was reviewed by a panel of mathematics lecturer from Royal University of Bhutan and mathematics teachers in Bhutan. Reliability of achievement test was measured by KR-20 which rendered the value 0.77. Pretest of 20 objective questions was administered before the experiment. Immediately after the treatment ended, posttest of the same questions was administered. Significant difference between the mean scores of pretest and posttest were tested at 0.05 levels by applying a t-test.

3.1 Data Analysis

The data gathered from the students pretest and posttest achievement test were analyzed using descriptive statistics and t-tests. The $p < 0.05$ level of significance was used to analyze the data to compare pretest and posttest achievement test of the students. Whereas the data gathered from the test at each lesson were represented in graph with the mean score.

4. RESULTS

4.1 Achievement Test Results

The Table 1 represented the mean scores of pretest and posttest of the students' achievement test. As shown in the table the mean score of the pretest ($M=6.66$; $SD=2.44$) and the posttest ($M=13.78$; $SD=3.02$). The mean was increased by 7.12 with the $p\text{-value}=0.00$ indicating that the students performed significantly better after the treatment with Polya's Problem-solving strategy.

Table 1. Pretest and posttest mean comparison

	Mean	SD	N	Mean difference	t-value	df	p-value
Pretest	6.66	2.44	32	7.12	16.477	31	.00
Posttest	13.78	3.02	32				

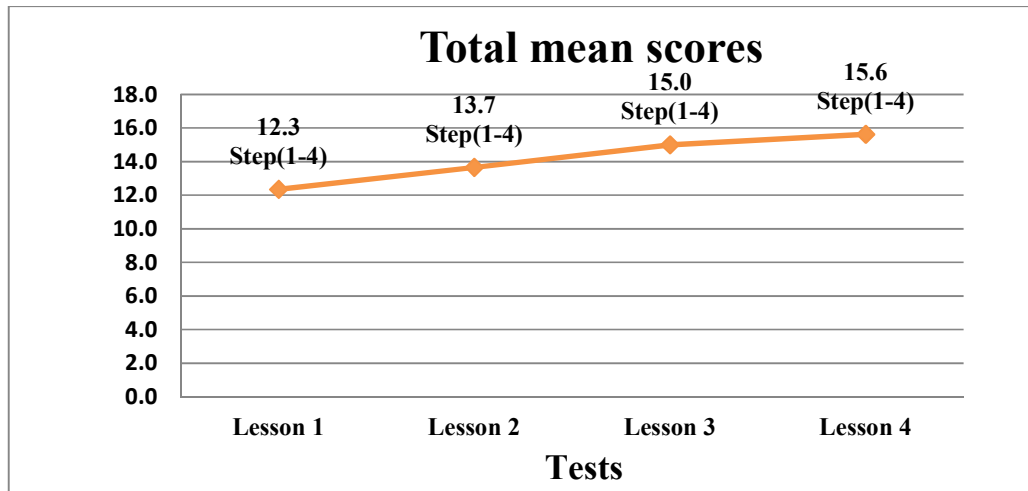


Fig. 1. Level of analyzing ability in all the steps of Polya's problem-solving model across the lessons

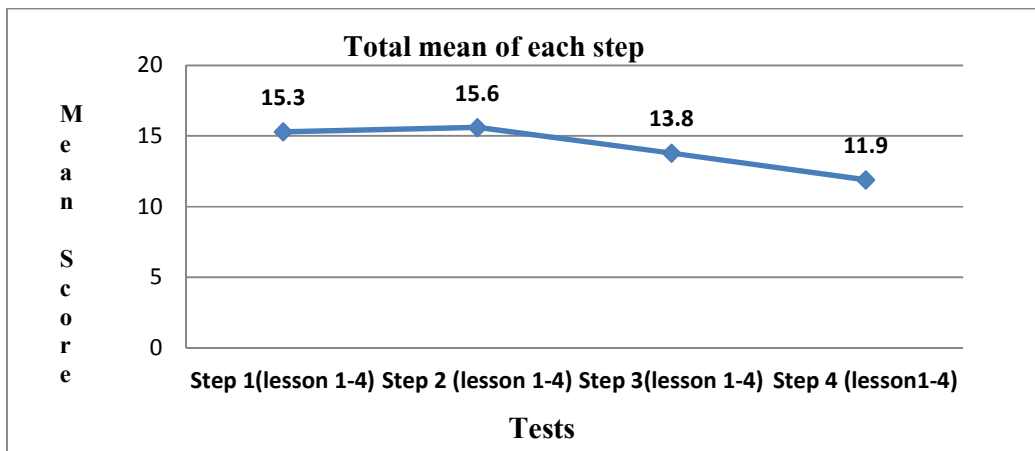


Fig. 2. Total mean of analyzing ability in each step of Polya's problem-solving model

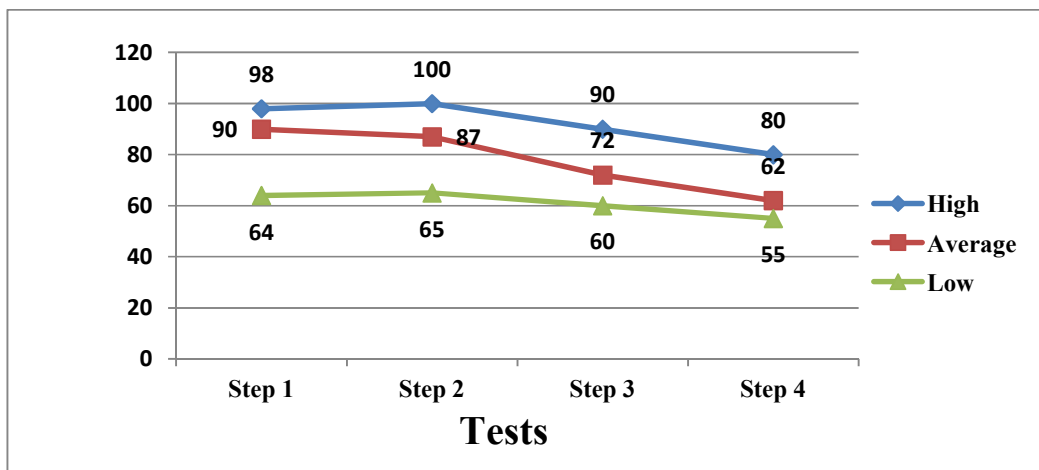


Fig. 3. Subtotal mean raw scores of analyzing ability in each step by different groups

4.2 Level of Analytical Ability across the Steps of PPSM in Each Lesson

The Fig. 1 presented the trend of total mean scores of students' performance, implementing step 1 (Understand the problem), step 2 (Devise a plan), step 3 (Carry out) and step 4 (Look back) computed from four lessons (sub-test). The total mean scores of all the steps of Problem-Solving Model, obtained from each test as shown in Figure indicated that students' analytical ability gradually increased from lesson 1 to lesson 4 lesson 1 (M=12.3), lesson 2 (M=13.7), lesson 3 (M=15.0), lesson 4 (M=15.6). This indicated that the students were able to analyze the question, devise a plan, carry out the plan and check their work for error after the repeated practice of PPSM. Fig. 2 showed the total mean scores of each step of Polya's Problem-Solving Model obtained from all the tests conducted at the end of the lessons. The records of the tests were maintained to see the implementation of analyzing skills with the use of the model. It was demonstrated from the figure that the mean scores of the first two steps of the Problem-Solving Model were high, indicating higher application of analyzing skills. It means that students were able to analyze the question, identify known and unknown information provided and make a plan. While in the last two steps the mean scores were low, indicating low application of analyzing skills. This showed that though the students were able to understand the question and make a plan, they were still lacking the ability to implement their plan correctly and failed to analyze each step to check for error.

4.3 Comparison of Analytical Ability across Different Ability Group

Fig. 3 given above illustrated the subtotal mean raw scores of different ability groups on application of analytical skills using Polya's Model. The raw scores of the students, obtained from the tests were used to group the students into high, average and low ability groups. Five students from each group were randomly selected and used for the comparison. It was clear from the figure that all the groups (High, Average and Low) could apply the first step (Understand the problem) and second step (Devise a plan). This indicated that the students could identify the problem and make a plan. While in the last two steps (Carry Out and Look Back) all the groups had difficulty in carrying out the plan and checking their own work. This

showed low application of analyzing skills. From the result it could be concluded that the different ability groups were able to perform better in the first two steps when compared to the last two steps.

5. DISCUSSION

The overall results of the study revealed that Polya's Problem-Solving Model increased the learning achievement and improved analyzing skills of fourth grade students at mathematics. The table above presented the mean difference of pretest and posttest. The mean was increased by 7.12, indicating that the students performed significantly better after the treatment with PPSM. The result was supported by the findings of Ali [2], Perveen [1] and Uche [8], who had also conducted similar studies. They concluded that there was significant gain between the pretest and posttest in teaching students with problem-solving model.

The findings of the result (Table 1) showed that the mean score of the pretest was 6.66 (below average) and posttest was 13.78 (above average). The reasons to account for such high achievement could be because of PPSM that guided the students throughout the process in solving the problem effectively and the other reason could be due to the instant feedback provided in the course by the teacher.

The result of the time series record (Fig. 1) revealed that in the beginning, students faced difficulty in applying analyzing skills with the use of the Problem-Solving Model but with much practice in groups, pairs and sometimes individually, all the students could apply the analyzing skills in all the steps correctly while solving the problem. The progression in application of Problem-Solving process supported the findings that the students not only performed better in mathematics but also used analyzing skills. The total mean scores of each step of Problem-Solving Model (Fig. 2) and the subtotal mean raw scores of different ability groups (Fig. 3) presented that as a whole, the students were able to apply analyzing skills in the first two steps compared to that of the last two steps. This finding is in line with finding of the study by Gray [10] on the impact of the application of Polya's first two steps to problem-solving have on an advanced high school mathematics classroom. The study demonstrated the posttest mean score difference was higher than the difference of mean scores on the pretest

and all the practice problems. The results established the first two steps of problem-solving had a positive impact on the students in the advanced math class.

The reason for high analyzing ability could be because of group discussion, and guidance of the model. In addition to that the researcher also asked questions and inspired the students to explain their understanding of the given questions. Other reason could be because, students need not put in lots of analyzing skills in the second step (Planning) as the methods were already prescribed in the text. The reason for low analyzing ability in step 3 (Carry out) and step 4 (Look back) could be because of the new methods introduced to solve division problems. Students did have the prior knowledge on the topic but they were not familiar with the prescribed new method in the new mathematics text. The present fourth grade students were dealing with the text for the first time. Besides, the students had limited ideas on the strategies to check for errors because the responsibility was assumed to be the teacher's work. Above all, the students of this stage fell under concrete operational stage where they can think logically but requires concrete concept, as their brains are not well-developed to engage themselves into higher level of thinking. However, the researcher helped the students by organizing enriching activities and provided blocks and counters to let them get hands on learning experiences with the new methods. Students were also guided in checking their errors by explaining the strategies such as the usage of multiplication tables and repeated addition.

6. CONCLUSION AND RECOMMENDATION

The study was carried out in one of the primary schools of Trongsa district in Bhutan with the sample of 32 fourth grade students. Data collected were analyzed using descriptive statistics and t-test. The result showed that the students' achievement level significantly improved with the inclusion of the PPSM with significant value .00 ($p < 0.05$). The quantitative data from the time series records also revealed that with the use of PPSM, it enhanced the analyzing ability of the students. From the students' reflection it was noted that they learnt more from group discussion which enabled them to perform better at individual level. It was also noted that Polya's Problem-Solving Model was useful and interesting though the students

encountered some difficulties in the course. Based on the findings, the following recommendations were made:

- 1) The finding of the study would be very useful for mathematics teacher in Bhutan to improve their teaching strategies.
- 2) Since the study revealed that the students had difficulty in applying analyzing skill in the last two steps of Polya's Model, a research could be conducted using Polya's Model with more emphasis on the last two steps and find out the factors affecting their performance. It would be interesting to conduct a comparative study of Polya's Problem-Solving Model with another problem-solving model and observe the differences in learning achievement and application of analyzing skill on lower grade students in mathematics.
- 3) The study will serve as reference for future research to carry out studies on similar fields.

7. LIMITATION

This study was conducted to examine the effectiveness of use of PPSM on learning achievement and analyzing ability in mathematics of fourth grade students. Generalizing the findings of this study may not be appropriate to other mathematics students in the country as the findings of this study are limited to the students of the focused class only. Moreover, this was the first kind of study using PPSM in Bhutanese mathematics classroom to test the effectiveness of the model. The sample taken was also minimal (32). Considering the limitation on the size of the sample. This study seeks the future study to examine the use PPSM as instructional methods in teaching mathematics problem solving at different grade levels that will serve as basis for further research in this field. Furthermore, it would be interesting to conduct a comparative study of PPSM with another problem solving model and observe the difference in mathematics learning achievement of the students.

CONSENT

Approval to the data collection was secured through the school principal. Parents' consent and students' assent forms were obtained before the task administered. Moreover, authorization

from the academic head of the school and class teacher were also obtained.

ETHICAL APPROVAL

The procedure used in this study has satisfied the ethical requirements defined by the APA Manual 6th Edition, and the study was conducted in compliance with School Action Research Committee (SARC).

COMPETING INTEREST

Authors have declared that no competing interests exist.

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