Asian Journal of Advanced Research and Reports

1(1): 1-8, 2018; Article no.AJARR.41109

Analytical Approach to Critical Diameters in Raindrop Size Distribution in Durban, South Africa

Oluwumi Adetan[1*] and Olumuyiwa Oludare Fagbohun[1]

[1]Department of Electrical and Electronic Engineering, Faculty of Engineering, Ekiti State University, Ado Ekiti, Nigeria.

Authors' contributions

This work was carried out in collaboration between both authors. Author OA designed the study, gathered and sorted the data, performed the mathematical analysis and wrote the first draft of the manuscript. Author OOF managed the analyses of the study, managed the literature searches and the references used in the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJARR/2018/v1i113014
Editor(s):
(1) Shu-Lung Kuo, Associate professor, Engineering Consultant, Kelee Environmental Consultant Corporation, Kaohsiung City, Taiwan and Department of Technology Management, the Open University of Kaohsiung, Kaohsiung City, Taiwan.
Reviewers:
(1) Lerato Shikwambana, South African National Space Agency, South Africa.
(2) Moumi Pandit, Sikkim Manipal Institute of Technology, India.
Complete Peer review History: http://www.sciencedomain.org/review-history/24300

Original Research Article

Received 13th February 2018
Accepted 19th April 2018
Published 24th April 2018

ABSTRACT

Adequate information of the raindrop size distribution is very significant for the prediction and evaluation of attenuation signal due to rain. In this study, an analytical approach is adopted to determine the peak diameter D_p where the specific rain attenuation is maxima in Durban (29º52'S, 30º58'E), South Africa; using the spherical raindrop shape at temperature T = 20ºC. The overall rainfall attenuation is computed by integrating over all the drop sizes and determine the differential change in the attenuation as observed over a fixed diameter interval, dD_i(= 0.1 *mm*). The critical diameters are the range of diameters where the rain attenuation is highly predominant, which constitutes the surface area under the curve and along the abscissa regions. The critical diameters are seen to coalesce around the peak diameter, at which the maximum attenuation occurs. The maximum specific rain attenuation peaks at the diameter $D_p = exp[\sigma^2(\alpha - 1) + \mu]\ (mm)$ It was observed that the peak diameter is frequency dependent while the parameters, μ, the mean and σ the standard deviation which determines the width of the distribution are found to be region-dependent. The peak attenuation for the stratiform rainfall type varies between 0.8 ≤ D ≤ 1.5 *mm*

**Corresponding author: E-mail: oadetan@gmail.com;*

whereas for the convective rainfall, the specific rain attenuation peaks between 1.4 ≤ D ≤ 2.7 *mm* at all frequencies. A proper knowledge of the rainfall attenuation characteristics is useful for proper planning and for the purpose of link budget analysis by operators in this particular region.

Keywords: Peak diameters; link budget analysis; critical diameters and specific rainfall attenuation.

1. INTRODUCTION

Generally, attenuation on radio propagation paths is caused by various atmospheric components such as gases, water vapour, clouds and rain. Rain attenuation, caused by scattering and absorption by the water droplet is one of the most important signal impairments influencing the attenuation of microwave (3-30 *GHz*) and millimetre wave (30 -300 *GHz*) [1]. Studies have shown that the effects of rainfall attenuation become significant and disturbing in satellite communications at a frequency above 10 GHz [2-4]. At this frequency, rain attenuation always causes outages, unavailability and poor quality of service [5]. The study of drop size distribution (DSD) is, however, vital for several application areas such as satellite meteorology, microwave communications, cloud physics and soil erosion [6]. The drop size distribution is an important parameter for the estimation of attenuation due to rain at microwave and millimetre-wave frequency applications because it governs all the microwave and rainfall integral relation. It has been established that modelling of DSD in tropical (such as Nigeria, Rwanda and Singapore) and temperate regions is not the same. This is due to the presence of heavy rainfall in tropical regions compared to temperate regions. When compared to temperate regions where a large database exists, measurements of drop size distributions in tropical regions are few. The negative exponential function, as proposed by Marshall and Palmer [7] or the *Laws and Parsons*, [8] and the gamma distribution model often characterize modelling of raindrop size distributions in the temperate region [9]. However, there is so much uncertainty in the preponderance and estimation of small diameter raindrops due to a limitation in the sensitivity of the measuring equipment. These models grossly overestimate the concentration of the small diameter raindrops in the tropical regions hence the Ajayi and Olsen [9] model was proposed and found suitable for the modelling of tropical rain drop size distributions and equally adequate for the determination of the specific attenuation.

In Durban, South Africa, some research work has been carried out on rainfall attenuation and DSD [10-15] establishing the suitability of the lognormal and gamma models for DSD modelling in the region. Similarly, various methods and models have been adopted by some researchers across the globe to investigate the particular contributions of certain raindrop diameters to the specific rain attenuation [16-20]. The influence of critical raindrop diameters on the specific rainfall attenuation in Durban, South Africa using randomly selected rainfalls analyzed in this study. Different rain rate values representing strati-form and convective rain types were selected for the purpose of analysis over the measured raindrop size distribution. The three-parameter lognormal DSD model as determined in [15] for Durban, South Africa is used to representing the measured DSD, N(D).

The critical raindrop diameters are the range of diameters over which the contribution to the rainfall attenuation is predominant. Different researchers within the tropical and temperate regions have investigated the particular contribution of certain diameters to the rainfall attenuation. Notable among them are Lee et al. and Lakmish et al. in Singapore [16,17] Fiser (in the Czech Republic [18] Lam et al. in Malaysia [19] and Marzuki et al. in Equatorial Indonesia [20]. In this study, an analytical approach is adopted to determine the peak diameter D_p where the specific rain attenuation is maxima in Durban (29º52'S, 30º58'E), South Africa; using the spherical raindrop shape at temperature *T* = 20ºC.

2. MATERIALS AND METHODS OF DATA COLLECTION

The Joss-Waldvogel (J-W) RD-80 disdrometer located at the roof top of the Electrical, Electronic and Computer Engineering building, University of Kwa-Zulu Natal, Durban (29º52'S, 30º58'E), South Africa was used to obtain data samples used in this work.

The measurements were taken over a period of three (3) years in Durban at a sampling time of 60 seconds. The disdrometer used in this work is capable of measuring the rainfall rate, reflectivity, rainfall accumulation and the raindrop size distribution with an accuracy of ±5%. Rain events (rainfall occurrence in minutes) with overall sum of drops less than 10 were discarded from the data samples to compensate for the dead- time errors as they are considered to be within the noise level [21]. The location is shielded from noise and abnormal winds. Very minimal equipment outage was observed during the period of data collection at this location.

Fig. 1 Joss-Waldvogel RD-80 impact disdrometer system connected to a personal computer [RD-80 product information: http://www.disdromet.com/1_index_e.htm].

The integral rain parameters of interest in this work are computed directly from the disdrometer spectra and represented by [21]:

$$P = a\sum_{n=1}^{20} C_n X_n \quad (1)$$

where P is the rainfall accumulation, a is the product of parameters that are invariant with drop size, C_n is the number of drops in the nth channel, and X_n is the moments. Generally, the measured rain drop size distribution $N(D_i)$ from the disdrometer data is the number of raindrops per cubic meter per millimeter diameter ($mm^{-3}\ m^{-1}$) as adopted by [22, 23] and computed as:

$$N(D_i)\ \frac{n_i}{v(D_i)\times T\times A\times dD_i}(m^{-3}mm^{-1}) \quad (2)$$

where n_i is the number of drops measured in the drop size bin, $v(D_i)$ is the terminal velocity of Gun Kinzer's [22] water drops in m/s, T is the one-minute sampling time in 60 s, A is the cross sectional sampling area of the disdrometer given as 50cm^2 and dD_i is the representative change in diameters interval of the bin in mm. The data was sorted and classified into different types of rain based on rainfall rates R in millimeter per hour according to [24,25] as: drizzle, (0 $mm/h \le R <$ 5 mm/h), widespread, (5 $mm/h \ge R <$ 10 mm/h), shower, (10 $mm/h \le R <$ 40 mm/h) and thunderstorm ($R>$ 40 mm/h). About 100,000 data samples were obtained from the disdrometer over a period of 3 years. The minimum and maximum rain rates obtained were 0.003 mm/h and 117.15 mm/h. In this work, the three-parameter lognormal DSD model was used.

The lognormal distribution model is expressed by [12]:

$$N(D) = \frac{N_T}{\sqrt{2\pi}\times\sigma\times D} exp\left[-\frac{1}{2}\left(\frac{\ln(D)-\mu}{\sigma}\right)^2\right] \quad (3)$$

where N_T (concentration of rainfall drops) is a function of climate, geographical location of measurements and rainfall type, μ is the mean of $\ln(D)$ and σ is the standard deviation which determines the width of the distribution. The three parameters in (3) above are related to the rainfall rate R by [22]:

$$N_T = \quad a_0 R^{b_0} \quad (4)$$

$$\mu = A_\mu + B_\mu \ln R \quad (5)$$

$$\sigma^2 = A_\sigma + B_\sigma \ln R \quad (6)$$

where a_0,b_0,$A_{\boldsymbol{\mu}}$, $B_{\boldsymbol{\mu}}$, A_σ and B_σ are coefficients of moment regression determined using the least squares method of regression technique. The three-parameter lognormal DSD model as determined in [15] is given as:

$$\mu = -0.3104 + 0.1331\ \ln R \quad (7)$$

$$\sigma^2 = 0.0738 + 0.0099\ \ln R \quad (8)$$

$$N_T = 268.07R^{0.4068} \quad (9)$$

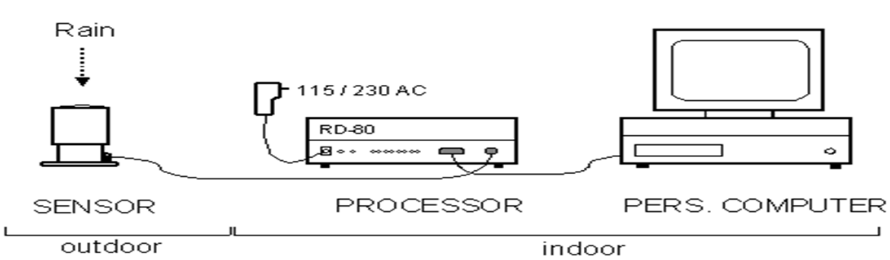

Fig. 1. Joss-Waldvogel RD-80 impact disdrometer system connected to a personal computer [RD-80 product information: http://www.disdromet.com/1_index_e.htm]

2.1 Mathematical Model of the Peak Diameter and the Specific Rain Attenuation

The specific attenuation γ of microwave signals due to rain is expressed as:

$$\gamma = \int_{d_0}^{d_{max}} Q_{ext}(D)N(D)dD \quad \left(\frac{dB}{km}\right) \tag{10}$$

where Q_{ext} is the extinction cross sections which are dependent on the drop diameter *D*, the wavelength λ, and the complex refractive index of water drop *m*, which in turn is a function of the frequency and temperature. The extinction cross section Q_{ext} is found by applying the classical scattering theory of Mie for a plane wave impinging upon a spherical absorbing particle. The Mie scattering theory is applied under the assumption that each spherical raindrop illuminated by a plane wave is uniformly distributed in a rain filled medium. Similarly, it is assumed that the distance between adjacent drops is large enough to avoid any interaction between them. The extinction cross sections given as:

$$Q_{ext}(D) = \kappa D^{\alpha} \tag{11}$$

N (D) is the drop size distribution model in m^{-3} mm^{-1} which in this case can be lognormal or gamma, *D* is the raindrop diameters in mm, κ and α are frequency dependent parameters. By substituting the expressions for the extinction cross section and drop size distribution (DSD) model (lognormal DSD model in this case) of equations (11) and (3) into (10), the differential specific rain attenuation can be maximized as:

$$\Delta\gamma = \kappa D^{\alpha} * \frac{N_T}{\sigma D\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left[\frac{\ln(D)-\mu}{\sigma}\right]^2\right\} \Delta D \tag{12}$$

$$\Delta\gamma = \kappa D^{\alpha-1} * \frac{N_T}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left[\frac{\ln(D)-\mu}{\sigma}\right]^2\right\} \Delta D \tag{13}$$

At the peak diameter *D*, $\frac{\Delta\gamma}{\Delta D} = 0$ and ΔD is fixed and constant,

$$(\alpha-1)D^{\alpha-2} - D^{\alpha-1}\left[\frac{1}{\sigma D}\frac{(\ln D-\mu)}{\sigma}\right]^2 = 0 \tag{14}$$

$$(\alpha-1) = \frac{1}{\sigma^2}(\ln D - \mu) \tag{15}$$

It follows from equation (15) therefore,

$$D_p = exp[\sigma^2(\alpha-1)+\mu] \tag{16}$$

The maximum specific rain attenuation peaks at the diameter, D_p (in mm) given by the expression (16). The Mätzler's MATLAB [26] functions are used for the estimation of k and α. Table 1 shows the computed values of k and α of the power law relation in equation (11) at frequencies of 10 to 100 GHz.

Table 1. Values of κ and α at f = 10-100 GHz at T = 20ºC

Frequencies (*GHz*)	K	α
10	0.3857	4.5272
25	2.4567	4.0186
40	4.3106	3.5077
60	6.0493	3.0094
80	7.0623	2.6621
100	7.6874	2.4156

3. RESULTS AND DISCUSSION

Tables 2 and 3 show the state-form and convective rain types with their respective critical diameters. The estimated values of μ and σ in equations (7) and (8) have been used for the purpose of analysis while α is a frequency dependent parameter as provided in Table 1. It can be observed that the peak diameters D_p at which the differential attenuation is maximum increases as the rain rate increases with varying input parameters. As the operating frequency increases, the peak diameter, D_p at which the specific rain attenuation is maximum decreases. The peak attenuation for the strati-form rainfall type varies between $0.8 \le D \le 1.5$ *mm* while the convective rainfall specific rain attenuation peaks between $1.4 \le D \le 2.7$ *mm* at all frequencies. The input parameters μ and σ in equations (7) and (8) used in Table 2 are regime-dependent. The peak diameter D_p is frequency-dependent since α is frequency dependent.

The three-parameter lognormal DSD model was also used to estimate the parameters required to investigate the drop sizes which produce a major contribution to the total specific rainfall attenuation for the selected rain rate values (strati-form and convective rainfall types). The attenuation created by drops in the diameter intervals $0.1 \ge D \ge 7.0$ *mm* at various frequencies of transmission is shown in Table 4. The total specific attenuation increases with increasing frequencies and rain rate. The highest and prevailing contribution to the specific attenuation occurs at $D \sim 2\,mm$ for the stratiform and convective rain types as shown in Figs. 2 and 3. The total percentage fraction formed by drops in the diameter range 0.5 *mm* ≤ *D* ≤ 2.5 *mm* and 1.0 mm ≤ *D* ≤ 3.0 *mm* are found to be most critical for the specific rain attenuation for the strati-form and convective rainfall types. It was observed that the contributions of larger diameters to the total attenuation is insignificant and can be negligible when compared to the medium and smaller diameters. Hence, the diameter ranges 0.5 *mm* ≤ *D* ≤ 2.5 *mm* are critical to attenuation in Durban being a coastal region characterized by the strati-form rain types.

Table 2. Rainfall regimes and the peak diameters for maximum attenuation at 10-100 *GHz* in Durban for Strati-form rainfall $(0 \leq R \leq 10\ mm/h)$

Strati-form					
Rain rate (*mm/h*)	**10 *GHz***	**25 *GHz***	**40 *GHz***	**60 *GHz***	**100 *GHz***
1	0.9371	0.9018	0.8677	0.8356	0.7990
2	1.0489	1.0056	0.9640	0.9250	0.8806
2.5	1.0877	1.0415	0.9972	0.9557	0.9085
3	1.1204	1.0718	1.0251	0.9816	0.9321
3.5	1.1488	1.0981	1.0494	1.0040	0.9524
4	1.1740	1.1214	1.0709	1.0238	0.9704
5.5	1.3335	1.2823	1.2328	1.1864	1.1333
7	1.4131	1.3589	1.3066	1.2575	1.2014
8.5	1.4805	1.4239	1.3692	1.3178	1.2592
9	1.5010	1.4436	1.3882	1.3362	1.2767

Table 3. Rainfall regimes and the peak diameters for maximum attenuation at 10-100 *GHz* in Durban for Convective rain $(R \geq 10\ mm/h)$

Convective					
Rain rate (*mm/h*)	**10 *GHz***	**25 *GHz***	**40 *GHz***	**60 *GHz***	**100 *GHz***
10	1.6418	1.5733	1.5074	1.4458	1.3756
15	1.8636	1.7841	1.7076	1.6361	1.5549
20	2.0389	1.9505	1.8655	1.7862	1.6961
30	2.3143	2.2117	2.1132	2.0214	1.9171
40	2.3886	2.2777	2.1715	2.0727	1.9608
60	2.5051	2.3846	2.2694	2.1625	2.0415
75	2.5716	2.4456	2.3252	2.2135	2.0874
85	2.6097	2.4805	2.3571	2.2427	2.1136
100	2.6600	2.5265	2.3992	2.2811	2.1480
120	2.7176	2.5792	2.4472	2.3250	2.1874

Table 4. Rainfall attenuation created by raindrops diameter range of 0.1*mm* ≥ D ≥ 7.0 *mm* using the lognormal model (frequency = 5-100 *GHz*)

Frequency (*GHz*)	R =1.4 *mm/h*	R= 14.2 *mm/h*	R= 44.5 *mm/h*	R= 77.7 *mm/h*
5	0.0057	0.0239	0.0754	0.2049
10	0.0433	0.2014	0.6899	2.0193
40	0.7903	2.8815	8.1216	20.0556
60	1.4570	4.7398	12.1929	27.8055
80	2.0836	6.2730	15.1661	32.7638
100	2.6307	7.5191	17.4093	36.2179

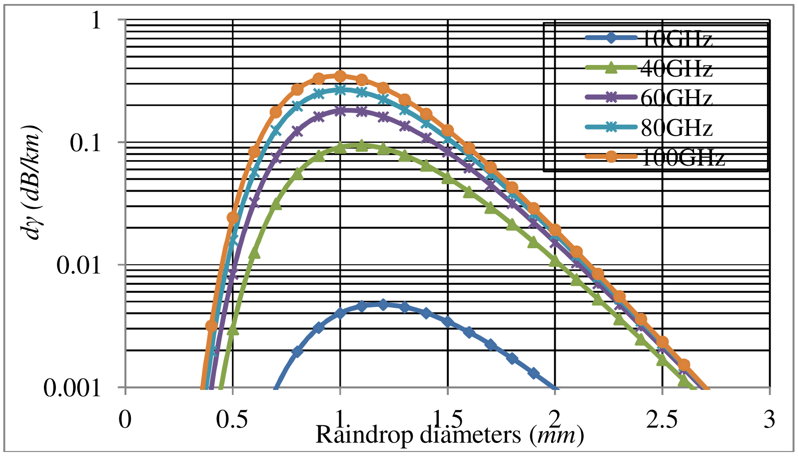

Fig. 2. Rainfall attenuation and raindrop diameters for the stratiform rainfall (R= 3.68 *mm/h*)

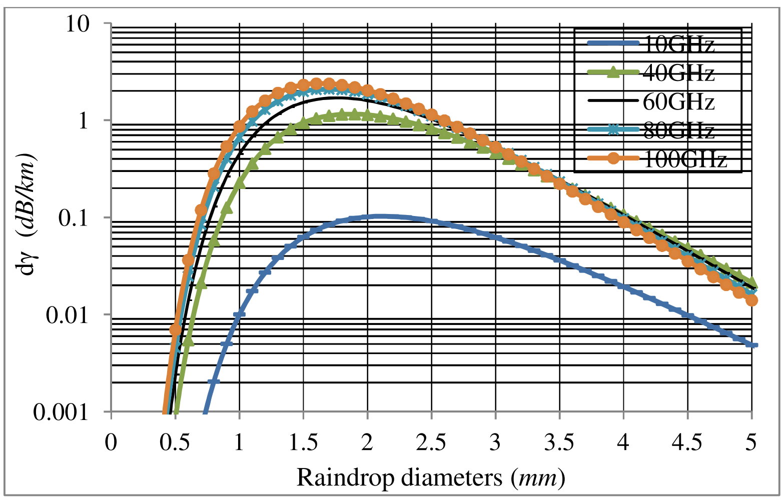

Fig. 3. Rainfall attenuation and raindrop diameters for the convective rainfall (R= 120 *mm/h*)

4. CONCLUSION

The lognormal DSD model parameters have been adopted to estimate the specific rain attenuation in this work. The strati-form and convective rainfall types were considered. The critical range of raindrop diameters over which the specific rainfall attenuation is at the maximum was investigated. The peak attenuation for the strati-form rainfall type varies between 0.8 ≤ D ≤ 1.5 *mm* whereas for the convective rainfall, the specific rain attenuation peaks between 1.4 ≤ D ≤ 2.7 *mm* at all frequencies. The results show a good agreement with earlier work done by [16-18] in the region. The critical diameters are seen to coalesce around the peak diameter, D_p at which the maximum attenuation occurs. The peak diameter was found to be $D_p = exp[\sigma^2(\alpha - 1) + \mu]\, mm$. It should be noted that D_p is frequency dependent, since α is dependent on frequency. The parameters, μ and σ are region-dependent. A good understanding of this rainfall attenuation characteristic will be helpful to properly design adequate fade margin levels, achieve the expected quality of service in a radio communication system operating in the South Africa region and for the purpose of link budget design by the engineers and service providers in this particular area.

ACKNOWLEDGEMENT

The authors will like to acknowledge Professor Thomas. J. Afullo of UKZN, South Africa for providing the disdrometer data used for this study.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Matthews PA, Radio wave propagation: VHF and above, Chapman and Hall Ltd.; 1965.
2. Ajayi GO, Ofoche EBC. Some tropical rainfall rate characteristics at Ile-Ife for microwave and millimetre wave applications. Journal of Applied Meteorology. 1984;23:562-67.
3. Chibiel J, Rahman A. Development of 1 min rain rate contour maps for microwave application in Malaysian Peninsula. Electronics Letters. 1999;35:1772-74.
4. Crane RK, Prediction of attenuation by rain. IEEE Transactions on Communications. 1980;28:1717-33.
5. Mawarni MY, Jafri D, Hong YL, Seat LJ. Analysis of inter-fade intervals at Ku-band in heavy rain region. IEEE International RF and microwave conference (RFM 2015); 2015.
6. Harikumar R, Sampath S, SasiKumar V. An empirical model for the variation of raindrop size distribution with rain rate at a few locations in southern India. Advances in Space Research. 2009;43: 837-44.
7. Marshall JS, Palmer WMK. The distribution of raindrops with size. Journal of Meteorology. 1948;5:165-166.
8. Laws JO, Parsons DA. The relation of raindrop-size to intensity. Transactions, American Geophysical Union. 1943;24: 452-60
9. Ajayi GO, Olsen RL. Modelling of a tropical raindrop size distribution for microwave and millimetre wave applications Radio Sci. 1985;20(2):193-202.
10. Adetan O, Afullo TJ. Comparison of two methods to evaluate the lognormal raindrop size distribution model in Durban. In proceedings of The Southern Africa Telecommunication Networks and Applications Conference (SATNAC). 2012; 2-5.
11. Owolawi P. Raindrop size distribution model for the prediction of rain attenuation in Durban. PIERS Online. 2011;7(6):516-23.
12. Alonge AA, Afullo TJ. Seasonal analysis and prediction of rainfall effects in Eastern South Africa at microwave frequencies. PIERS B. 2012;40:279-303.
13. Odedina MO, Afullo TJ. Determination of rain attenuation from electromagnetic scattering by spherical raindrops: Theory and experiment. Radio Science. 2010;45.
14. Afullo TJO. Raindrop size distribution modelling for radio link along the Eastern coast of South Africa. PIERS B. 2011;34: 345-66.
15. Adetan O, Afullo TJ. Three - parameter raindrop size distribution modelling for microwave propagation in South Africa. Proceedings of the International Association of Science and Technology for Development (IASTED) International Conference on Modeling and Simulation (Africa MS 2012). 2012;155-160.
16. Lee YH, Lakshmi S, Ong JT. Rain drop size distribution modelling in Singapore-critical diameters. In Proceedings of the Second European Conference on Antennas and Propagation (EUCAP). 2007;1-5.
17. Lakshmi S, Lee YH, Ong JT. The roles of particular raindrop size on rain attenuation at 11GHz. In Proceedings of 6th IEEE International Conference on Information, Communications & Signal Processing (ICICS). 2007;1-4.
18. Fiser O. The role of particular rain drop size classes on specific rain attenuation at various frequencies with Czech data example. Proceedings of ERAD. 2002; 113(116):113-117.
19. Lam HY, Din J, Luini L, Panagopoulos AD, Capsoni C. Analysis of raindrop size distribution characteristics in Malaysia for rain attenuation prediction. In General Assembly and Scientific Symposium, 2011 XXXth URSI. 2011;1–4.
20. Marzuki M, Kozu T, Shimomai T, Randeu WL, Hashiguchi H, Shibagaski Y. Diurnal variation of rain attenuation obtained from measurement of raindrop size distribution in Equatorial Indonesia. IEEE Transactions on Antennas and Propagation. 2009;57(4): 1191-96.
21. Tokay A, Kruger A Krajewski WF. Comparison of drop size distribution measurements by impact and optical disdrometer. Journal of Applied Meteorology. 2001;40:2083-97.
22. Gunn R, Kinzer GD. The terminal velocity of fall for water droplets in the stagnant air. Journal of Meteorology. 1949;6:243-48.

23. Bartholomew M. Disdrometer and tipping bucket rain gauge handbook," DOE/SC-ARM/TR-079, ARM Climate Research Facility; 2009.
24. Timothy KI, Ong JT, Choo EB. Raindrop size distribution using the method of moments for terrestrial and satellite communication applications in Singapore. Antennas and Propagation, IEEE Transactions on Communications. 2002; 50:1420-24.
25. Adimula IA, Ajayi GO. Variation in raindrop size distribution and specific attenuation due to rain in Nigeria. Annals of Telecommunications. 1996;51(2):87-93.
26. Mätzler C. MATLAB functions for Mie scattering and absorption, IAP Res. Rep; 2002.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history/24300