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On two-stage fuzzy random programming for water resources management

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In this paper, a two-stage fuzzy random programming for a management problem in terms of water resources allocation having fuzzy random variable coefficients and decision vector of random variables is studied. The first results show the fact that a fuzzy pseudorandom optimal solution of a two-stage fuzzy random programming may be resolved into a two of pseudorandom optimal solutions of relative two-stage random programmings. The subsequent results present a method that a fuzzy pseudorandom optimal solution of two-stage fuzzy random programming is structured by a two of pseudorandom optimal solutions of a relative two two-stage random programmings. A numerical example was given to clarify the obtained results.

Key words: Two-stage programming, fuzzy parameters, water resources management, interval analysis, fuzzy random variable, fuzzy approach.

INTRODUCTION

Stochastic programming deals with situations where some or all the parameters of a mathematical programming problem are described by stochastic variables rather than by deterministic quantities. Several models have been presented in the field of stochastic programming (Stancu-Minasian et al., 1976).

Contini (1978) developed an algorithm for stochastic goal programmings when the random variables are normally distributed with known means and variances. He transformed the stochastic problem into an equivalent deterministic quadratic programming problem, where the objective functions consisted of maximizing the probability of a vector of goals lying in the confidence region of a predefined size.

Teghem et al. (1986) and Leclercq (1982) have presented interactive methods in stochastic programming. Two major approach to stochastic programming as recognized by Goicoechea et al. (1982) and Kambo (1984) are:

1. Chance constrained programming,
2. Two-stage programming.

In fuzzy decision making problems, the concept of a maximizing decision was proposed by Bellman and Zadeh (1990). Fuzzy linear programming problem with coefficients was formulated by Negoita (1970) and called robust programming.

Dubois and Prade (1982) investigated linear fuzzy constraints. Tanaka and Asai (1984) also proposed a formulation of fuzzy linear programming with fuzzy coefficients. Wang and Wang (1985) built another theory

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of fuzzy programming on the basis of "Satisfaction degree" which has been used in engineering design. Wang and Qiao (1993) put forward some model linear programming with fuzzy random variable coefficients, and studied the solution and distribution problems of a fuzzy random programming. Over the past decades, the conflict-laden issues of water resources allocation among competing municipal, industrial and agricultural interests have been of increasing concerns (Huang and Chang, 2003; Wang et al., 2003). The competition among water users has been intensified due to growing population shifts, shrinking water availabilities, varying natural conditions, and deteriorating quality of water resources (Li and Huang, 2008).

The increased water demands and the inadequate water supplies have exacerbated the shortage of water resources; this has been considered as a major obstacle to sustainable water resources management. When the essential demand cannot be satisfied due to in sufficient resources, losses can hardly be avoided resulting in a variety of a diverse impact on Socio-economic development (Lu et al., 2010).

Wang and Adams (1986) proposed a two-stage optimization framework of planning reservoir operations, inflows were modeled as a periodic Markov processes. Eiger and Shamir (1991) developed a model for an optimal multi-period operation within a multi-reservoir system. Ferreto et al. (1998) examined a long-term by drothermal scheduling of multi-reservoir systems using a two-stage dynamic programming approach.

In many real-world problems, however, the quality and quantity of uncertain information is often not satisfactory enough to be presented as probabilities distribution. Even if such distributions for uncertain parameters are available, reflecting them in large-scale optimization models can be extremely challenging (Huang and Loucks, 2000).

Wang and Huang (2011) have also developed an interactive two-stage stochastic fuzzy programming approach through incorporating an interactive fuzzy resolution method within an inexact two-stage stochastic programming framework.

This paper will further be devoted to a model proposed in Wang and Huang (2011). Under a certain a condition we discuss the solution method of a fuzzy two-stage stochastic programming, which has fuzzy random variable coefficients and the decision vector of random variables. Ammar and Khalifa (2014) introduced a fuzzy programming approach for treating an interactive two-stage stochastic rough interval water resource management.

PRELIMINARIES

In order to discuss our problem conveniently, we shall state some necessary results on interval arithmetic, fuzzy numbers and fuzzy random variables. Moore (1979) introduced the concept of closed interval numbers.

Let $I(R)=\{[a^L, a^U]\}$, $a^L, a^U \in R=(-\infty, \infty), a^L \le a^U\}$ denote the set of all closed interval numbers on *R*.

Definition 1:

Suppose that $[a^L, a^U], [b^L, b^U] \in I(R)$.

We define :

1. $[a^L, a^U](+)[b^L, b^U]=[a^L+b^L, a^U+b^U]$.

2. $[a^L, a^U](-)[b^L, b^U]=[a^L-b^L, a^U-b^U]$.

3. $[a^L, a^U](\cdot)[b^L, b^U]=[\min(a^L\cdot b^L,\ a^L\cdot b^U,\ a^U\cdot b^L, a^U\cdot b^U,$
$\max(a^L\cdot b^L,\ a^L\cdot b^U,\ a^U\cdot b^L, a^U\cdot b^U)]$
$=[a^L\cdot b^L \wedge a^L\cdot b^U \wedge a^U\cdot b^L \wedge a^U\cdot b^U,$
$=[a^L\cdot b^L \vee a^L\cdot b^U \vee a^U\cdot b^L \vee a^U\cdot b^U]$.

4. The order relation $"\le"$ in $I(R)$ is defined by $[a^L, a^U](\le)[b^L, b^U]$ if and only if $a\le b^L,\ a^U \le b^U$.

5. If $a_t \in I(R),\ t\in T$, where *T* is an index set, we define $\vee_{t\in T} a_t=\sup\{a_t : t\in T\}$; $\wedge_{t\in T} a_t=\inf\{a_t : t\in T\}$.

In particular, for $[a_t^L, a_t^U]\in I(R),\ t\in T$, we define:

$$\vee_{t\in T}[a_t^L, a_t^U]=[\vee_{t\in T} a_t^L,\ \vee_{t\in T} a_t^U];\ \wedge_{t\in T}[a_t^L, a_t^U]=[\wedge_{t\in T} a_t^L,\ \wedge_{t\in T} a_t^U].$$

Throughout this paper, $F_0(R)$ denote the set of all bounded closed fuzzy numbers (that is, compact on *R*, for any $f\in F_0(R),\ f$ satisfies:

1. There exists $x\in R$ such that $f(x)=1$;

2. For any $\alpha\in(0,1],\ f_\alpha=[f_\alpha^L, f_\alpha^U]$ is a closed interval number on *R*,

And observed that $R\subset I(R)\subset F_0(R)$.

Definition 2:

Assume that $*$ is an algebraic operation on *R*; $f\cdot g\in F_0(R)$.

The algebraic operation $*$ on $F_0(R)$ is defined by $(f*g)(z)=\vee_{z=x*y}(f(x)\wedge g(y))$.

where $*$ may be $"+", "-", "\cdot"$, etc.

The order relation $"\leq"$ on $F_0(R)$ is defined by $f \leq g$ if and

only if and only if $f_\alpha \leq g_\alpha$ for any $\alpha \in (0,1]$.

Definition 3:

Let $\{f_t,\ t \in T\} \subset F_0(R),\ \alpha \in (0,1]$.

(a) $\underset{t\in T}{\wedge}(f_t)_\alpha$,

(b) $\underset{t\in T}{\vee} f_t$ is defined by a fuzzy number $g \in F_0(R)$ such that $g_\alpha = \underset{t\in T}{\vee}(f_t)_\alpha$, where *T* is an index set.

Definition 4:

Let $f, g \in F_0(R)$ then for any $\alpha \in (0,1]$, we have $(f * g)_\alpha = f_\alpha * g_\alpha$,

where $*$ may be continuous algebraic operation (Qiao et al., 1994).

Definition 5:

Let (M, N, P) be a probability measure space.

A mapping $\tilde{a}: M \rightarrow F_0(R)$ is called a fuzzy random variable on (M, N), if for any $\alpha \in (0,1]\ \tilde{a}_\alpha(w) = \{\alpha : x \in R, \tilde{a}(w)(*)\alpha\} = [a_\alpha^L(w), a_\alpha^U(w)]$ is a random variables on (M, N) (Wang and Zhang, 1992). Denote the set of all fuzzy random variables on (M, N) by $FR(M)$ (where $w \in M$).
Moreover, let $*$ be an algebraic operation on $F_0(R)$.
The algebraic operation $*$ on $FR(M)$ may be defined by $(\tilde{a} * \tilde{b})(w) = \tilde{a}(w) * B(w)$ for any $w \in M$, and $\tilde{a} \leq \tilde{b}$ if and only if $\tilde{a}(w) \leq \tilde{b}(w)$ for any $w \in M$, where $\tilde{a}, \tilde{b} \in FR(M)$

If $\{\tilde{a}_t : t \in T\} \subset FR(M)$, then $(\underset{t\in T}{\wedge}\tilde{a}_t)(w) = \underset{t\in T}{\wedge}\tilde{a}_t(w)$, for any $w \in M$.

STATEMENT OF PROBLEM

A typical two-stochastic programming model for water resources management is (Wang and Huang (2011) as follows:

$$\max\ f = \sum_{i=1}^{m} NB_i T_i - E\left[\sum_{i=1}^{m} C_i S_{iQ}\right]$$

$$\text{s.t.}\ \sum_{i=1}^{m} (T_i - S_{iQ})(1+\delta) \leq Q$$

(Water availability constraints) (1)

$$S_{iQ} \leq T_i \leq T_{i\max}, \forall i$$

(Water- allocation target constraints)

$$S_{iQ} \geq 0, \forall i$$

(Non- negativity and technical constraints)

Where f = system benefit ($); $NB_i =$ net benefit to user i per m^3 of water allocated ($/$m^3$) (first- stage revenue parameters); $T_i =$ allocated target for water that is promised to user $i\,(m^3)$ (first- stage decision variables); $E[.] =$ expected value of a random variable ; $C_i =$ loss to user i per m^3 of water not delivered , $C_i > NB_i\ (\$/m^3)$ (second- stage cost parameters); $S_{iQ} =$ shortage of water to user i when the seasonal flow is $Q(m^3)$ (second – stage decision variables); $Q =$ total amount of seasonal flow (m^3) (random variables); δ_i = rate of water loss during transportation; $T_{i\max} =$ maximum allowable allocation amount for user $i\,(m^3); m =$ total number of water users; $i =$ water user , $i = 1,2,3,$ where $i = 1$ for the municipality, $i = 2$ for the industrial user, and $i = 3$ for the agricultural sector.
Based on the findings of Huang and Loucks (2000), the problem (1) can be reformulated as follows:

$$\max\ f = \sum_{i=1}^{m} NB_i\ T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i S_{ij}$$

s.t. $\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta) \le q_j, \forall j$ (2)

$S_{ij} \le T_i \le T_{i\max}, \forall i, j,$

$S_{ij} \ge 0, \forall i, j,$

Where S_{ij} denotes the amount by which the water-allocation target (T_i) is not met when the seasonal flow is q_j with probability p_j.

In this paper, we study the problem (2) with fuzzy random variable coefficients and decision vector of random variables as:

Model 1: (two-stage fuzzy random programming)

Find (S, T) to max $f = \sum_{i=1}^{m}\tilde{N}B_i\, T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j \tilde{C}_i S_{ij}$ subject to: $\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta) \le q_j, \forall j$

$S_{ij} \le T_i \le T_{i\max}, \forall i, j, \quad S_{ij} \ge 0, \forall i, j,$

Where, $\tilde{N}B_i, \tilde{C}_i, \tilde{\delta}, q_j \in FR(M)$.

The solution method of Model 1 will be discussed under the condition $\tilde{N}B_i \ge 0, \tilde{C}_i \ge 0$, and $\tilde{\delta} \ge 0$.

Model 1 may be rewritten as follows:

Model 2: Find (S^*, T^*) from $\tilde{G}$ such that

$$\sum_{i=1}^{m}\tilde{N}B_i\, T^*{}_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i \tilde{C}_i S^*{}_{ij} = \max_{(S,T)\in\tilde{G}} \left(\sum \tilde{N}B_i\, T_i^* - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j \tilde{C}_i S_{ij}^* \right),$$

where

$$\tilde{G} = \{(S,T) : \sum_{i=1}^{m}(T_i - S_{ij})(1+\tilde{\delta}) \le \tilde{q}, \quad j = 1, ..., m;$$
$$S_{ij} \le T_i \le T_{i\max}, i = 1,2,\ldots,m; S_{ij} \ge 0, i = 1,2,\ldots,m; j = 1,2,\ldots,n\}.$$

Definition 6:

The (S^*, T^*) which satisfy the condition in Model 2, is called a fuzzy random optimal solution of Model 2.

Observe that Where, $\tilde{N}B_i, \tilde{C}_i, \tilde{\delta}, q_j \in FR(M)$, for any $\alpha \in (0, 1]$ their α-level cuts are random values denoted by $(\tilde{N}B_i)_\alpha = [(NB_i)_\alpha^L, (NB_i)_\alpha^U], (\tilde{C}_i)_\alpha = [(C_i)_\alpha^L, (C_i)_\alpha^U], (\tilde{\delta})_\alpha = [(\delta)_\alpha^L, (\delta)_\alpha^U]$, and $(\tilde{q}_j)_\alpha = [(q_j)_\alpha^L, (q_j)_\alpha^U]$, $I = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.$

Corresponding to Model 2, we structure the following programming problems:

Model 2.1: Find (S^*, T^*) from $\tilde{G}_\alpha$ such that

$$\sum_{i=1}^{m}(NB_i)_\alpha^L T_i^* - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j (c_i)_\alpha^U S_{ij}^*$$

$$= \max_{(S,T)\in\tilde{G}_\alpha^U} \left(\sum_{i=1}^{m}(NB_i)_\alpha^L T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j (c_i)_\alpha^U S_{ij} \right)$$

where

$$\tilde{G}_\alpha^U = \{(S,T) : \sum_{i=1}^{m}(T_i - S_{ij})(1+(\delta)_\alpha^U) \le (q_j)_\alpha^U$$

$$S_{ij} \le T_i \le T_i^{\max}, \qquad i = 1, ..., m;\ j = 1, ..., n;$$

$$S_{ij} \ge 0, \qquad i = 1, ..., m;\ j = 1, ..., n\},$$

for any $\alpha \in (0, 1]$.

Model 2.2: Find (S^*, T^*) from $\tilde{G}_\alpha$ such that

$$\sum_{i=1}^{m}(NB_i)_\alpha^U T_i^* - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j (c_i)_\alpha^L S_{ij}^*$$

$$= \max_{(S,T)\in\tilde{G}_\alpha^L} \left(\sum_{i=1}^{m}(NB_i)_\alpha^U T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j (c_i)_\alpha^L S_{ij} \right)$$

for any $\alpha \in (0, 1]$.

For any given $\alpha \in (0, 1]$ Models 2.1-2.2 are linear programming problems with random variable coefficients; we may use the simple method discussed in Wang and Qiao (1993) to find random optimal solutions of the two models.

Lemma 1.

Let $\tilde{\delta} \ge 0$. If $(S, T) \in \tilde{G}$, then $(S^L, T^L) \in \tilde{G}_\alpha^L$, $(S^U, T^U) \in \tilde{G}_\alpha^U$ for any $\alpha \in (0, 1]$.

Proof:

$(S,T)\in\tilde{G}$ implies $\sum_{i=1}^{m}(T_i - S_{ij})(1+\tilde{\delta}) \le \tilde{q}_j,\ j=1,\dots,n$; $S_{ij} \le T_i \le T_i^{\max}$, $i=1,\dots,m;\ j=1,\dots,n\};\ S_{ij}\ge 0,\ i=1,\dots,m;\ j=1,\dots,n\}$

It follows that from Definitions 2 and 5:

$$\sum_{i=1}^{m}(T_i - S_{ij})(1+(\tilde{\delta})_\alpha) = \sum_{i=1}^{m}(T_i - S_{ij})(1+[\delta_\alpha^L,\ \delta_\alpha^U])$$
$$= \sum_{i=1}^{m}[(T_i - S_{ij})(1+\delta_\alpha^L),\ (T_i - S_{ij})(1+\delta_\alpha^U)]$$
$$= \left[\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^L),\ \sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^U)\right].$$

Thus we obtain

$$\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^L) \le (q_j^L)_\alpha, \qquad j=1,\dots,n,$$
$$\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^U) \le (q_j^U)_\alpha, \qquad j=1,\dots,n,$$

for every $i=1,\dots,m;\ j=1,\dots,n$.

Therefore, $\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^U) \le (q_j^L)_\alpha$, and $\sum_{i=1}^{m}(T_i - S_{ij})(1+\delta_\alpha^L) \le (q_j^L)_\alpha$.

That is to say, $(S^L, T^L)\in\tilde{G}_\alpha^L,\ (S^U,T^U)\in\tilde{G}_\alpha^U$. ϒ

Theorem 2.

Assume that

$$\tilde{\delta}\ge 0,\ \tilde{N}B_i \ge 0, \tilde{C}_i \ge 0, \tilde{G}_\alpha^L \subset \left\{(S^L,T^L):(S,T)\in\tilde{G}\right\}$$

and $\tilde{G}_\alpha^U \subset \{(S^U,T^U):(S,T)\in\tilde{G}\}$ where $\alpha\in(0,1]$.

If (S^*,T^*) is a random optimal solution of Model 2, then for any $\alpha\in(0,1]$, we have:

(1) (S^{*L},T^{*L}) is a random optimal solution of Model 2.2,

(2) (S^{*U},T^{*U}) is a random optimal solution of Model 2.1,

(3) $W_\alpha^L =$ max $\left(\sum_{i=1}^{m}(\tilde{N}B_i)_\alpha^L T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j(\tilde{C}_i)_\alpha^U S_{ij}\right),\ (S,T)\in\tilde{G}_\alpha^U,$

$W_\alpha^U =$ max $\left(\sum_{i=1}^{m}(\tilde{N}B_i)_\alpha^U T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j(\tilde{C}_i)_\alpha^L S_{ij}\right),\ (S,T)\in\tilde{G}_\alpha^L,$

Where

$\tilde{W} =$ max $\left(\sum_{i=1}^{m}(NB_i T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i S_{ij}\right),\ (S,T)\in\tilde{G},\ \tilde{W}_\alpha = [W_\alpha^L, W_\alpha^U].$

Proof:

Suppose that (S^*,T^*) is a fuzzy random optimal solution of Model 2, then $(S^*,T^*)\in\tilde{G}$, and

$$\sum_{i=1}^{m} N B_i T_i^* - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j \tilde{c}_i S_{ij}^* = \max_{(S,T)\in\tilde{G}}\left(\sum_{i=1}^{m} N B_i T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j \tilde{c}_i S_{ij}\right)$$

Since $\tilde{\delta}\ge 0$, by using Lemma 1, we can see that

$(S^{*L},T^{*L})\in\tilde{G}_\alpha^L,\ (S^{*U},T^{*U})\in\tilde{G}_\alpha^U$,

$\tilde{G}_\alpha^L = \{(S^L,T^L):(S,T)\in\tilde{G}\}$

and

$\tilde{G}_\alpha^U = \{(S^U,T^U):(S,T)\in\tilde{G}\}$.

It follows by Theorem 1, and Definition 1, and observing the conditions $\tilde{N}B_i \ge 0$, $\tilde{C}_i \ge 0$, $\tilde{\delta}\ge 0$ that is

max $\sum_{i=1}^{m}(\tilde{N}B_i)_\alpha T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j(\tilde{C}_i)_\alpha S_{ij}), (S,T)\in\tilde{G}$

=max$\left(\sum(\tilde{N}B_i)_\alpha^L T_i - \sum\sum p_j(\tilde{C}_i)_\alpha^U S_{ij},\ \sum(\tilde{N}B_i)_\alpha^U T - \sum\sum p_j(\tilde{C}_i)_{\alpha_i}^L S_{ij}\right)$,

$(S,T)\in\tilde{G}$, this leads to (3) of the Theorem 2 is true.

Numerical example

Consider the Model 1 with the following economic data illustrated in Table 1. At $\alpha = 0.5$.

The solution of Model 2.2 is

$T_1 = 3,\quad T_2 = 4,\quad T_3 = 4,\quad S_{11} = S_{12} = S_{13} = S_{21} = S_{22} = S_{23} = S_{33} = 0$,

$S_{31} = 9,\ S_{32} = 3.8,\ p^L = 373.6$ $.

The solution of Model 2.1 is

$T_1 = 1,\ T_2 = 2, T_3 = 7, S_{11} = S_{12} = S_{13} = S_{22} = S_{23} = S_{33} = S_{31} = 0, S_{21} = 7.9365, S_{32} = 7.4348,$

$p^U = 560.6519$ $.

The solution of Model 2 is

$T_1 = [1, 3],\ T_2 = [2, 4],\ T_3 = [4, 7],\ S_{11} = S_{12} = S_{13} = S_{22} = S_{23} = S_{33} = 0$

$S_{21} = [0,\ 7.9365],\ S_{31} = [0,\ 9],\ S_{32} = [7.4348,\ 3.8]$,

Table 1. Model 1 economic data.

Activity	User		
	Municipal $(i=1)$	**Industrial** $(i=2)$	**Agricultural** $(i=3)$
Maximum allowable allocation $T_{i\max}$	8	8	8
Water-allocation target T_i	[1, 3]	[2, 4]	[4, 7]
Net benefit when water dema-nd is satisfied $\tilde{NB}_i$	(80, 90, 100)	(50, 60, 70)	(30, 31, 32)
Reduction of net benefit when demand is not delivered $\tilde{C}_i$	(220, 240, 260)	(70, 80, 90)	(40, 50, 60)

Flow level	**Probability** p_j (%)	**Seasonal flow** $\tilde{q}_j$
Low $(j=1)$	0.2	(2, 3, 4)
Medium $(j=2)$	0.6	(8, 10, 12)
High $(j=3)$	0.2	(16, 18, 20)
Water loss $\tilde{\delta}$	-	(0.1, 0.2, 0.3)

and $f=[f^L, f^U]=[373.6, 560.6519]$$.

Conclusions

In this paper, a two-stage fuzzy random programming for water resources management having fuzzy random variable coefficients and decision vector of random variables has been presented. The results have shown that the fuzzy random optimal solution of the problem under consideration has relative two-stage random programmings.

Conflict of Interest

The author has not declared any conflict of interest.

REFERENCES

Ammar EE, Khalifa HA (2014). Interactive two- stage stochastic fuzzy rough programming for water resources management. J. Adv. Phy, 4(3):622-642.

Bellman RE, Zadeh LA (1990). Decision making in a fuzzy environment. Manage. Sci. 17:141-164.

Contini B (1978). A stochastic approach to goal programming. Oper. Res. 16:576-586.

Dubois D, Prade H (1982). System linear fuzzy constraints. Fuzzy Sets Syst. 13:1-10.

Eiger G, Shamir U (1991). Optimal operation of reservoirs by stochastic programming. Eng. Optim. 17:293-312.

Ferreto RW, Rivera JF, Shahidehpaur SM (1998). A dynamic programming two-stage algorithm for long-term hydrothermal scheduling of multi reservoir systems. IEEE Trans. Power Syst. 13:1534-1540.

Goicoechea A, Hansen DR, Duckstein L (1982). Multiobjective Decision Analysis with Engineering and Business applications (Wiley, New York).

Huang GH, Chang NB (2003). The perspectives of environmental informatics and systems analysis. Environ. Inform. 1:1-6.

Huang GH, Loucks DP (2000). An inexact two-stage stochastic programming model for water resources management under uncertainty. Civ. Eng. Environ. Syst. 17:95-118.

Kambo NS (1984). Mathematical Programming Techniques (Affliated East-West Pross Pvt. Ltd)

Leclercq JP (1982). Stochastic programming: an interactive multi criteria approach. Eur. J. Oper. Res. 10:33-41.

Li YP, Huang GH (2008). Interval parameter resources management under uncertainty. Water Res. Manage. 22:681-698.

Lu HW, Huang GH, He L (2010). Development of an interval-valued fuzzy linear programming method based on infinite α-cuts for water resources management. Environ. Model. Softw. 25:354-361.

Moore RE (1979). Method and applications of Interval Analysis (SIAM, Philadelphia, PA).

Negoita CV (1970). Fuzziness in Management, OPSA/TIMS, Miami.

Qiao Z, Zaban Y, Wang G (1994). On fuzzy random linear programming. Fuzzy Sets Syst. 65:31-49.

Stancu-Minasian I, Wels MJ (1976). A research bibliography in stochastic programming. Oper. Res. 24:1078-1119.

Tanaka H, Asai K (1984). Fuzzy linear programming problem with fuzzy numbers. Fuzzy Sets Syst. 13:1-10.

Teghem J Jr., Dufrance D, Thauvoye M, Kunch P (1986). An interactive method for multi-objective linear programming under uncertainty. Eur. J. Oper. Res. 26:65-82.

Wang G, Wang W (1985). Fuzzy optimum design of structure. Eng. Optim. 8:291-300.

Wang D, Adams BJ (1986). Optimization of real-time reservoir operations with Markov decision processes. Water Res. 22:345-352.

Wang G, Zhang Y (1992). The Theory of fuzzy stochastic processes, Fuzzy Sets Syst. 51:161-178.

Wang G, Qiao Z (1993).Convergence of sequences of fuzzy random variables and its application. Fuzzy Sets Syst. 59:295-311.

Wang LZ, Fang L, Hipel KW (2003). Water resources allocation: A cooperative game theoretic approach. J. Environ. Inform. 2:11-22.

Wang S, Huang GH (2011). Interactive two-stage stochastic fuzzy programming for water resources management. J. Environ. Manag. 92:1986-1995.