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A Predator-Prey Model with Logistic Growth for Constant Delayed Migration

Apima Bong'ang'a Samuel[1*]

[1] *Department of Mathematics and Statistics, Kaimosi Friends University College, Kenya.*

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

Predator prey models predict a broad range of results depending on characteristics of predators, prey and the environment in which they interact. The environment in which these species live in and interact is usually made up of many patches, and these patches are connected via migration. The instantaneous migration of these species from one patch to another may not be realistic since there may be barriers during migration such as a busy infrastructure through the natural habitat. A predator-prey model, with logistic growth for both species and constant delayed migration, is developed and analyzed in this paper. It is shown that these species will survive if they migrate at higher rates in search of sustaining resources. Thus, for the species to coexist, we recommend that factors that slow down migration rates should be addressed, for example, reducing human activities and settlement in natural habitat.

Keywords: Predator-prey model; delayed migration; symmetric; asymmetric.

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[*] *Corresponding author: E-mail: apimabs@gmail.com;*

1 Introduction

The interaction between the prey species which acts as a source of food to the predator species, can be described by a predator-prey model [1, 2, 3]. The Lotka-Volterra model is an example of a classical predator-prey model which represents the interaction of these species in a homogeneous (single patch) environment [2]. Most classical predator-prey models assume that the environment will always be homogeneous, this may not be realistic since the environment is heterogeneous (many patches) and its connected by a diffusion-like process referred to as migration [4, 5, 6, 7, 8, 9]. Migration takes place when a species transfers from one patch to another due to some hostile situations in its initial patch, for example, ecological factors like floods, human settlement and activities in the natural habitat like logging, intraspecific competition, predation, among others [10, 11, 12]. Migration can either be constant or variable. If the number of species migrating per unit time is a constant portion of the species density in a given patch then the migration is constant, otherwise the migration is variable [8, 9, 13].

An assumption in the Lotka-Volterra model is that the prey population will grow unhindered in the absence of predation. This may not be realistic because other factors such as diseases and intraspecific competition may affect the species population apart from predation [8, 14]. Thus incorporating a parameter which is determined by the available sustaining resources, known as a carrying capacity, makes the model to be more realistic [2, 14].

Another assumption in most predator prey models, for example Abdllaoui et al. [5] and Mchich et al. [6], is that these species will migrate instantly after facing some hostile situations. In reality, after a species encounters a hostile situations, the species may meet a barrier, like a busy infrastructure passing through the natural habitat or a swollen river, while trying to migrate. With these barriers, a delayed migration of the prey and predator species may occur [7, 8, 9, 15].

2 The Model

The model uses the framework of the predator-prey model with logistic growth for both species as given in Murray [2]. There is a constant delayed migration for both species and the two patches are coupled via migration. The model is given by,

$$\begin{aligned}
\dot{n}_1(t) &= \alpha_1(n_2(t-\tau)-n_1(t)) + n_1\left(r_1(1-\frac{n_1}{K}) - \frac{A_1p_1}{n_1+B_1}\right),\\
\dot{n}_2(t) &= \alpha_2(n_1(t-\tau)-n_2(t)) + n_2\left(r_2(1-\frac{n_2}{K}) - \frac{A_2p_2}{n_2+B_2}\right),\\
\dot{p}_1(t) &= \beta_1(p_2(t-\tau)-p_1(t)) + kp_1\left(1-\frac{hp_1}{n_1}\right)\\
\dot{p}_2(t) &= \beta_2(p_1(t-\tau)-p_2(t)) + kp_2\left(1-\frac{hp_2}{n_2}\right) \qquad (2.1)
\end{aligned}$$

Where $n_i = n_i(t)$ is the prey population at time t in patch i and $p_i = p_i(t)$ is the predator population at time t in patch i. The intrinsic growth rate of the prey population is denoted by r_i , the prey carrying capacity and the predator growth rate are denoted by K and k, respectively. The constant h is the predator's mortality rate. Predation parameters are denoted A and B. The predator migration rate and the prey migration rate are denoted by β_i and α_i respectively, $i, j = 1, 2$, $i \neq j$. The parameter τ denotes a time delay in the migration of the prey and predator species. In the model given in equation 2.1, the predator grows logistically unlike in Samuel et al. [8] where the predator density is only affected by the prey density.

3 Model Analysis

Let $Y_i(t) := \Big(n_i(t), p_i(t)\Big)$, $i = 1, 2$ then Equation (2.1) becomes,

$$\dot{Y}_i(t) \quad = \quad (Y_j(t-\tau) - Y_i(t))\gamma + f_i(Y_i(t)), \quad i, j = 1, 2, i \neq j, \tag{3.1}$$

where, $\gamma = \big(\alpha\beta^T$ and

$$\begin{aligned} f_i(Y_i(t)) = \Bigg(n_i \Bigg(r_i(1 - \frac{n_i}{K}) - \frac{A_i p_i}{n_i + B_i}\Bigg) \\ k p_i \Bigg(1 - \frac{h p_i}{n_i}\Bigg). \end{aligned}$$

Let $Y(t) := (Y_1(t), Y_2(t))$ and $f(Y(t), Y(t-\tau))$ represent the vector field on the right hand side of Equation (3.1), thus Equation (3.1) becomes,

$$\dot{Y}(t) = f(Y(t), Y(t-\tau)). \tag{3.2}$$

Let $C = \mathcal{C}([-\tau, 0], \mathbb{R}^4)$ be a Banach space equipped with the sup norm, $\|\phi\| = sup|\phi(\theta)| \leq r, (0 \leq r < \infty)$, for $\theta \in [-\tau, 0]$ and where $|\phi(\theta)|$ denotes a Euclidean norm of $\phi(\theta)$.

Let the initial condition be given by,

$$\varphi(t) := Y(t) \mid_{[-\tau,0]}, \tag{3.3}$$

where $\varphi \in C$. Since $f(Y(t), Y(t-\tau)) \in \mathcal{C}(\mathbb{R}^4 \times C, \mathbb{R}^4)$, Equation (3.2) subject to Equation (3.3) has a unique solution. For more on existence and uniqueness of solutions, see for instance Hale and Lunel [16].

We wish to exploit the symmetries in the migration terms of the linear part of Equation (3.1), given by;

$$\dot{Y}_i(t) \quad = \quad (Y_j(t-\tau) - Y_i(t))\gamma, \quad i, j = 1, 2, i \neq j, \tag{3.4}$$

which can be symbolized by two invariant manifolds. We will examine the long term behaviors of these two invariant manifolds. To find these invariant manifolds, first, we show that an exponential estimate of the solution of the Equation (3.4) is bounded. Then we obtain the characteristic equation so that we can show that Laplace Transform of Equation (3.4) exists. Lastly, an application of the Laplace transform methods in complex variables to the terms describing migration in Equation (3.1) is required.

3.1 Exponential boundedness

Lemma 3.1. *The solution of Equation (3.4) subject to the initial condition in Equation (3.3) for* $t \geq 0$*, satisfies,*

$$\mid Y(t) \mid \leq (1 + A\tau)e^{2At} \mid \varphi \mid, \tag{3.5}$$

where $\mid . \mid$ *denotes a sup norm in* $\mathbb{R}$ *as well as a matrix norm.*

The proof of Lemma (3.1) can be obtained in Samuel et al. [8].

3.2 Characteristic equation of the coupling terms

To obtain solutions of Equation (2.1), we assume a solution of the form

$$Y(t) = e^{\lambda t}c, \tag{3.6}$$

where $c \in \mathbb{R}^4$ with c a nonzero 4 by 1 column vector. Substituting Equations (3.6) into the linear part of Equation (2.1), we obtain the characteristic equation

$$\lambda I_4 e^{\lambda t}c = \begin{pmatrix} -\alpha e^{\lambda t} & 0 & \alpha e^{\lambda(t-\tau)} & 0 \\ 0 & -\beta e^{\lambda t} & 0 & \beta e^{\lambda(t-\tau)} \\ \alpha e^{\lambda(t-\tau)} & 0 & \alpha e^{\lambda(t-\tau)} & 0 \\ 0 & \beta e^{\lambda(t-\tau)} & 0 & -\alpha e^{\lambda t} \end{pmatrix} c \tag{3.7}$$

where I_4 denotes the identity matrix of order 4.

We obtain the following characteristic equation from equation (3.7)

$$((\beta+\lambda)^2 - \beta^2 e^{-2\lambda\tau})((\alpha+\lambda)^2 - \alpha^2 e^{-2\lambda\tau}) = 0 \tag{3.8}$$

3.3 Invariant manifold

Taking the Laplace transform of Equation (3.4), we get

$$\begin{pmatrix} -\lambda I_2 - I_2\gamma & e^{-\lambda\tau}I_2\gamma \\ e^{-\lambda\tau}I_2\gamma & -\lambda I_2 - I_2\gamma \end{pmatrix} \begin{pmatrix} Y_1(\lambda) \\ Y_2(\lambda) \end{pmatrix} = \begin{pmatrix} Y_1(0) \\ Y_2(0) \end{pmatrix}. \tag{3.9}$$

where $\gamma = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$.

Equation (3.9) is symmetric in nature. On simplifying equation (3.9), adding (respectively subtracting) the set of equations involving $Y_2(0)$ to (respectively from) $Y_1(0)$ in equation (3.9), we obtain

$$(-I_2\lambda - I_2\gamma + I_2\gamma e^{-\lambda\tau})(Y_1(\lambda) + Y_2(\lambda)) = Y_1(0) + Y_2(0), \tag{3.10}$$

$$(-I_2\lambda - I_2\gamma - I_2\gamma e^{-\lambda\tau})(Y_1(\lambda) - Y_2(\lambda)) = Y_1(0) - Y_2(0), \tag{3.11}$$

The Matrix $(-I_2\lambda - I_2\gamma + I_2\gamma e^{-\lambda\tau})^{-1}$ is non-singular when λ is such that $(-I_2\lambda - \gamma + \gamma e^{-\lambda\tau}) \neq 0$ and $(-\lambda - \gamma - \gamma e^{-\lambda\tau})^{-1} I_2$ is non-singular when λ is such that $(-\lambda - \gamma - \gamma e^{-\lambda\tau}) \neq 0$, thus the inverse Laplace transformfor Equation (3.10) and Equation (3.11) is

$$\begin{aligned} (Y_1(\lambda) + Y_2(\lambda)) &= \mathcal{L}^{-1}\{(-\lambda - \gamma + \gamma e^{-\lambda\tau})I_2)^{-1}\}(Y_1(0) + Y_2(0)), \\ (Y_1(\lambda) - Y_2(\lambda)) &= \mathcal{L}^{-1}\{(-\lambda - \gamma - \gamma e^{-\lambda\tau})I_2)^{-1}\}(Y_1(0) - Y_2(0)), \end{aligned} \tag{3.12}$$

We have two manifolds; when $Y_1(t) = -Y_2(t)$ we have the asymmetric manifold and when $Y_1(t) = Y_2(t)$ we have the symmetric manifold.

The two linear subspaces of $\mathbb{R}^4$ are defined by,

$$\begin{aligned} \Theta &= \{(Y_1(t), Y_2(t)), Y_i(t) \in \mathbb{R}^2 : Y_1(t) - Y_2(t) = 0\}, \\ \Pi &= \{(Y_1(t), Y_2(t)), Y_i(t) \in \mathbb{R}^2 : Y_1(t) + Y_2(t) = 0\}. \end{aligned} \tag{3.13}$$

A change of coordinates, introduced to help in examining Equation (2.1), is defined by,

$$\begin{aligned} n_1 &:= u_1 + u_2,\ p_1 := v_1 + v_2, \\ n_2 &:= u_1 - u_2,\ p_2 := v_1 - v_2. \end{aligned} \tag{3.14}$$

With the notion that the prey and predator species are of similar type irrespective of the patch, we take $B_1 = B_2 := B$, $b_1 = b_2 := b$, $s_1 = s_2 := s$, $a_1 = a_2 := a$, $A_1 = A_2 := A$, $r_1 = r_2 := r$, $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. Using the transformation in Equation (3.14) in Equation (2.1), we obtain,

$$\begin{aligned}
\dot{u}_1 &= \alpha(u_1(t-\tau) - u_1) + \frac{r}{K}\Big(Ku_1 - u_1^2 - u_2^2\Big) - A\Big(\frac{Bu_1v_1 + u_1^2v_1 - u_2(-u_2v_1 + Bv_2)}{(u_1+B)^2 - u_2^2}\Big), \\
\dot{v}_1 &= \beta(v_1(t-\tau) - v_1) - k\Big(\frac{-u_1^2v_1 + u_2v_1(-2hv_2 + u_2) + hu_1(v_1^2 - v_2^2)}{u_1^2 - u_2^2}\Big), \\
\dot{u}_2 &= -\alpha(u_2(t-\tau) + u_2) + \frac{r}{K}\Big(Ku_2 - 2u_1u_2\Big) - A\Big(\frac{Bu_2v_1 + u_1(B+u_1)v_2 - u_2^2v_2}{(u_1+B)^2 - u_2^2}\Big), \\
\dot{v}_2 &= -\beta(v_2(t-\tau) + v_2) + k\Big(\frac{-u_2^2v_2 + u_1v_2(-2hv_1 + u_1) + hu_2(v_1^2 + v_2^2)}{u_1^2 - u_2^2}\Big).
\end{aligned} \tag{3.15}$$

where $u_i = u_i(t)$ and $v_i = v_i(t)$, $i = 1, 2$. The linear subspace in Equation (3.13) becomes

$$\Pi = \{(u_1, v_1, 0, 0) \in \mathbb{R}^4 : (u_1, v_1) \in \mathbb{R}^2\},$$

$$\Theta = \{(0, 0, u_2, v_2) \in \mathbb{R}^4 : (u_2, v_2) \in \mathbb{R}^2\}.$$

On both Π and Θ the system reduces to two dimensional systems of the form

$$\dot{U}_1 = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} U_1(t-\tau) + \begin{pmatrix} -\alpha + r - \frac{ru_1}{K} & \frac{-Au_1}{u_1+B} \\ 0 & -\beta + k(1 - \frac{hv_1}{u_1}) \end{pmatrix} U_1 \tag{3.16}$$

and

$$\dot{U}_2 = \begin{pmatrix} -\alpha & 0 \\ 0 & -\beta \end{pmatrix} U_2(t-\tau) + \begin{pmatrix} -\alpha + r & \frac{-Au_2^2}{B^2-u_2^2} \\ 0 & -\beta + k - \frac{khv_2}{u_2} \end{pmatrix} U_2 \tag{3.17}$$

where $U_i = (u_i, v_i)^T, i = 1, 2$ respectively.

Next we examine the stability of solutions on the two manifolds, this will help us predict long-term behaviors of the solutions of Model (2.1).

3.4 Asymmetric manifold

On solving the system in equation (3.16), we let $U_1(t) = e^{\lambda t}C_1$, then we obtain the following characteristic equation from equation (3.16),

$$(r - \alpha - \lambda + \alpha e^{-\lambda\tau})(k - \beta - \lambda + \beta e^{-\lambda\tau}) = 0 \tag{3.18}$$

We will use the following theorem to show the nature of solutions for Equation(3.16)

Theorem 3.1. *Equation (3.16) has*

- *(i) a sink at the origin for $\beta > k$ and $\alpha > r$,*
- *(ii) a saddle at the origin for either $\beta > k$ and $\alpha < r$ or $\beta < k$ and $\alpha > r$,*
- *(iii) a source at the origin for $\beta < k$ and $\alpha < r$,*
- *(iv) a periodic solution $\beta = k$ and $\alpha = r$*

Proof. Using the first factor of equation (3.18), we have

$$(r - \alpha - \lambda + \alpha e^{-\lambda\tau}) = 0 \tag{3.19}$$

Let

$$z = (\lambda + \alpha - r)\tau \tag{3.20}$$

Then equation (3.19) becomes

$$z = \alpha\tau e^{-z} e^{(-r+\alpha)\tau} \tag{3.21}$$

The following lemma which is found in [17] will be used to simplify equation (3.21)

Lemma 3.2. *The equation $z = be^{-z}$ has simple pure imaginary roots,*
$z = i(\pi/2 + 2m\pi)$, *for* $b = -(\pi/2 + 2m\pi)$
$z = 0$, *for* $b = 0$
$z = i(\pi/2 + (2m+1)\pi)$, *for* $b = (\pi/2 + (2m+1)\pi)$
where $m = 0, 1, 2, ...$ *and there are no other purely imaginary roots.*

Using Lemma (3.2) where $b = \alpha\tau e^{-(r-\alpha)\tau} > 0$, we get $z = i(\pi/2 + (2m+1)\pi$ for $\alpha\tau e^{-(r-\alpha)\tau} = (\pi/2 + (2m+1)\pi)$. Equation (3.20) becomes

$$\lambda = \frac{i(\pi/2 + (2m+1)\pi)}{\tau} - \alpha + r \tag{3.22}$$

Therefore

(i) for $\alpha < r$, equation (3.22) has roots with positive real parts,

(ii) for $\alpha > r$, equation (3.22) has roots with negative real parts,

(iii) for $\alpha = r$, equation (3.22) has purely imaginary roots.

For the second factor of equation (3.18),

$$(k - \beta - \lambda + \beta e^{-\lambda\tau}) = 0. \tag{3.23}$$

Let

$$z = (\lambda + \beta - k)\tau \tag{3.24}$$

From Lemma (3.2), equation (3.24) becomes

$$\lambda = \frac{i(\pi/2 + (2m+1)\pi}{\tau} - \beta + k \tag{3.25}$$

Therefore

(i) for $\beta < k$, equation (3.24) has roots with positive real parts,

(ii) for $\beta > k$, equation (3.24) has roots with negative real parts,

(iii) for $\beta = k$, equation (3.24) has purely imaginary roots.

The results for equation (3.18) therefore becomes

(i) a saddle at the origin for either $\beta < k$ or $\alpha < r$ or source for $\beta < k$ and $\alpha < r$, that means that either the predator population will become extinct which will lead to the prey population growing in a logistic manner or the prey population become extinct which makes the predator population becoming extinct after some time due to their lack of food.

(ii) a sink at the origin for $\beta > k$ and $\alpha > r$, that implies that the two species will coexist.

(iii) a center for $\beta = k$ and $\alpha = r$, that means that the population will oscillate.

□

3.4.1 Numerical analysis for asymmetric manifold

The following parameter values are used in simulating the graphs of equation (3.16), $u_1(0) = 160$, $v_1(0) = 120$, $B = 0.2$, $A = 0.2$, $h = 0.03$, $K = 250$ and $\tau = 0.1$. These parameter values are adapted from Samuel et al. [8]. The asymmetric manifold's simulations gives,

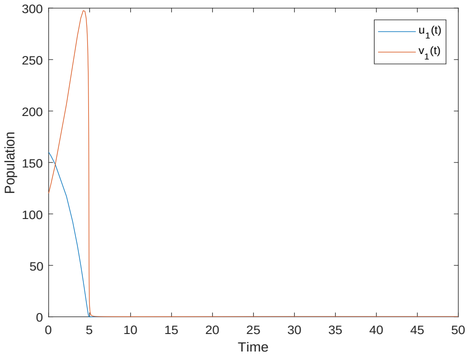

Fig. 1. $\alpha < r$, $\alpha = 0.1$ $\beta = 0.6$ $r = 0.2$ $k = 0.3$

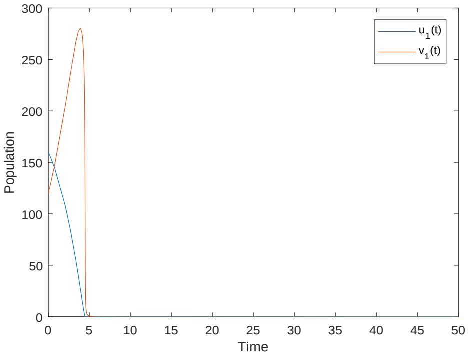

Fig. 2. $\beta < k$, $\alpha = 0.2$ $\beta = 0.2$ $r = 0.1$ $k = 0.3$

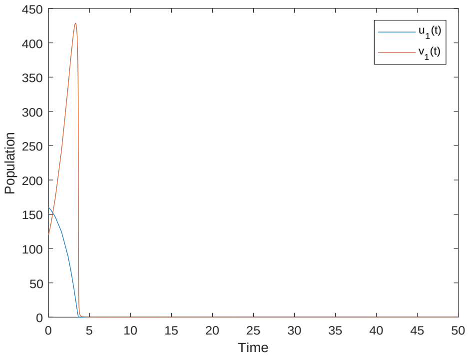

Fig. 3. $\alpha < r$ & $\beta < k$, $\alpha = 0.1$ $\beta = 0.3$ $r = 0.2$ $k = 0.5$

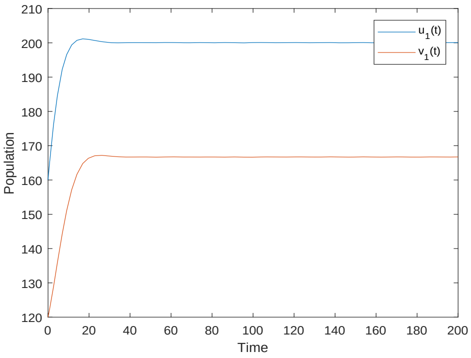

Fig. 4. $\alpha > r$ & $\beta > k$, $\alpha = 0.2$ $\beta = 0.2$ $r = 0.1$ $k = 0.1$

In Figs. 1-3, when either $\alpha < r$ or $\beta < k$ or both, the total predator density in the two patches will grow from the initial density, it will be greater than the total prey density in the two patches after some time t. As the predator density grows, this makes the prey density to go to extinction, this in turn makes the predator density to begin decreasing and finally going to extinction because of lack of food.

In Fig. 4, when $\alpha > r$ and $\beta > k$ then both densities coexist because the food source (the prey) will always be in plenty and the predator density does not increase, therefore it minimally affects the prey density. The total prey density in the two patches will always be greater than the total predator density in the two patches.

3.5 Symmetric manifold

Similarly, on solving the system in equation (3.17), let $U_2(t) = e^{\lambda t}C_2$, then the following characteristic equation is obtained from equation (3.17),

$$(r - \lambda - \alpha - \alpha e^{-\lambda\tau})(k - \lambda - \beta - \beta e^{-\lambda\tau}) = 0 \tag{3.26}$$

The following theorem is used to show the behavior of solutions for equation (3.17),

Theorem 3.2. *Equation (3.17) has*

- *(i) a sink at the origin for $\beta > k$ and $\alpha > r$,*
- *(ii) a saddle at the origin for either $\beta > k$ and $\alpha < r$ or $\beta < k$ and $\alpha > r$,*
- *(iii) a source at the origin for $\beta < k$ and $\alpha < r$,*
- *(iv) a periodic solution $\beta = k$ and $\alpha = r$*

Proof. Using the first factor of equation (3.26), we have

$$(r - \lambda - \alpha - \alpha e^{-\lambda\tau}) = 0 \tag{3.27}$$

Let

$$z = (\lambda + \alpha - r)\tau \tag{3.28}$$

Then equation (3.27) becomes

$$z = -\alpha\tau e^{-z}e^{(-r+\alpha)\tau} \tag{3.29}$$

Using Lemma (3.2) where $b = -\alpha\tau e^{(-r+\alpha)\tau} > 0$, we get $z = i(\pi/2 + 2m\pi)$ for $-\alpha\tau e^{(-r+\alpha)\tau} = -(\pi/2 + 2m\pi)$. Equation (3.28) becomes

$$\lambda = \frac{i(\pi/2 + 2m\pi)}{\tau} - \alpha + r \tag{3.30}$$

Therefore

- (i) for $\alpha < r$, equation (3.30) has roots with positive real parts,
- (ii) for $\alpha > r$, equation (3.30) has roots with negative real parts,
- (iii) for $\alpha = r$, equation (3.30) has purely imaginary roots.

For the second factor of equation (3.26),

$$(k - \lambda - \beta - \beta e^{-\lambda\tau}) = 0. \tag{3.31}$$

Let

$$z = (\lambda + \beta - k)\tau \tag{3.32}$$

From Lemma (3.2), equation (3.32) becomes

$$\lambda = \frac{i(\pi/2 + 2m\pi)}{\tau} - \beta + k \tag{3.33}$$

Thus,

- (i) for $\beta < k$ equation (3.33) has roots with positive real parts,
- (ii) for $\beta > k$, equation (3.33) has roots with negative real parts,
- (iii) for $\beta = k$, equation (3.33) has purely imaginary roots.

From equation (3.26),

(i) we have a saddle at the origin for either $\alpha < r$ or $\beta < k$ or source at the origin for $\alpha < r$ and $\beta < k$, that means the model becomes unstable. Therefore, either the predator population will become extinct which will lead to the prey population growing in a logistic manner or the prey population become extinct which makes the predator population becoming extinct after some time due to their lack of food.

(ii) we have a sink at the origin for $\alpha > r$ and $\beta > k$, that implies that the model is stable and therefore the two species will coexist.

(iii) we have a center at the origin for either $\alpha = r$ and $\beta = k$, that means a periodic solution occurs.

□

3.5.1 Numerical analysis for symmetric manifold

The following parameter values, adapted from Samuel et al. [8], are used in simulating the results of equation (3.17), $A = 0.2$, $B = 0.2$, $h = 0.03$, $\tau = 0.1$ $u_2(0) = 160$ and $v_2(0) = 120$. Simulations for the Symmetric manifold gives,

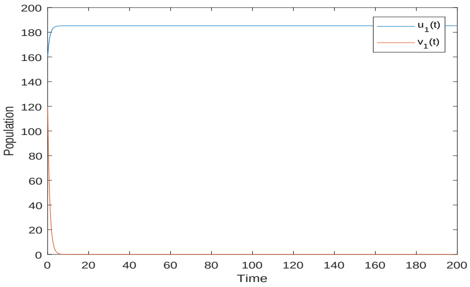

Fig. 5. $\alpha < r$, $\alpha = 0.1$ $\beta = 0.6$ $r = 0.2$ $k = 0.3$

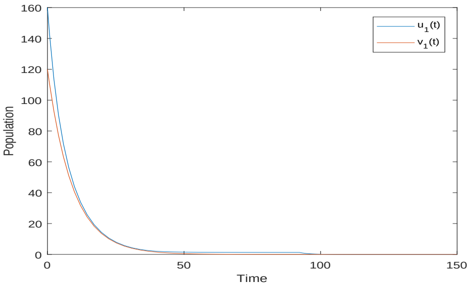

Fig. 6. $\beta < k$, $\alpha = 0.2$ $\beta = 0.2$ $r = 0.1$ $k = 0.3$

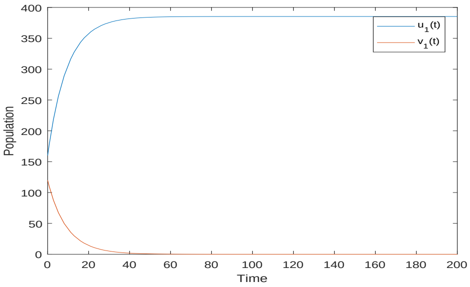

Fig. 7. $\alpha < r$ & $\beta < k$, $\alpha = 0.1$ $\beta = 0.3$ $r = 0.2$ $k = 0.5$

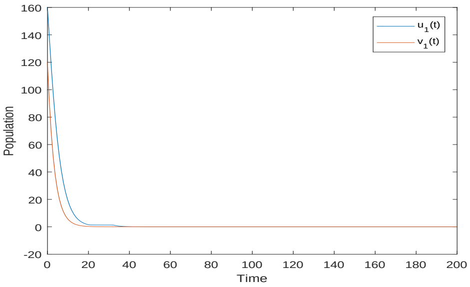

Fig. 8. $\alpha > r$ & $\beta > k$, $\alpha = 0.2$ $\beta = 0.2$ $r = 0.1$ $k = 0.1$

In Fig. 5 and Fig. 7, when $\alpha < r$ and $\alpha < r$ & $\beta < k$ respectively, the prey density grows bounded by the carrying capacity and the predator population becomes extinct after some time t. This shows that these simulations are for an unstable systems since the predator density becomes dinstinct.

In Fig. 6 and Fig. 8, when $\alpha > r$ and $\alpha > r$ & $\beta > k$ respectively, both populations stabilize at zero, and since this is the symmetric manifold, then the populations in both patches for the two species is the same. Therefore either the population is zero or the population will always be the same in the two patches showing that the densities will coexist. The reason the two species coexist in Fig. 6 is because in this case the prey are able to move at high rates after facing factors which may affect their existence, this helps the prey species to exist at all times thus providing a food source for the less mobile predator species. In Fig. 8, both species are able to migrate at high rates to a patch which contains their sustaining resources.

4 Conclusion

It is evident from this study that survival of the species depends on the migration rates as shown in the analysis of our formulated model where the species will survive if they migrate at higher rates in search of sustaining resources. Factors which may slow down migration of these species include human activities in the natural habitats like logging and natural causes like bad climatic conditions, limited food resources and overpopulation in a patch. For the species to coexist, we recommend that factors that slow down migration rates should be addressed, for example, reducing human activities and settlement in natural habitat.

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Competing Interests

Author has declared that no competing interests exist.

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