Performance Modelling of Health-care Service Delivery in Adekunle Ajasin University, Akungba-Akoko, Nigeria Using Queuing Theory

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Original Research Article

Abstract

The queuing theory is the mathematical approach to the analysis of waiting lines in any setting where arrivals rate of the subject is faster than the system can handle. It is applicable to the health care setting where the systems have excess capacity to accommodate random variation. Therefore, the purpose of this study was to determine the waiting, arrival and service times of patients at AAUA Health- setting and to model a suitable queuing system by using simulation technique to validate the model. This study was conducted at AAUA Health- Centre Akungba Akoko. It employed analytical and simulation methods to develop a suitable model. The collection of waiting time for this study was based on the arrival rate and service rate of patients at the Outpatient Centre. The data was calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modelled and simulated using the PYTHON software. The result obtained from the simulation model showed that the mean arrival rate of patients on Friday week1 was lesser than the mean service rate of patients (i.e. 5.33> 5.625 (λ > μ). What this means is that the waiting line would be formed which would increase indefinitely; the service facility would always be busy. The analysis of the entire system of the AAUA health centre showed that queue length increases when the system is very busy. This work therefore evaluated and predicted the system performance of AAUA Health-Centre in terms of service delivery and propose solutions on needed resources to improve the quality of service offered to the patients visiting this health centre.

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1 Introduction

It is a goal universally acknowledged that a healthcare system should treat its patients and especially those in need of critical care – in a timely manner. However, this is often not achieved in practice, particularly in public healthcare systems that suffer from high patient demand and limited resources Au-Yeung et al., 2006; [1].

The service facilities whose patients vary generally in capacity and size, from a small outpatient clinic to a large, urban hospital to referral hospital. Regardless of these differences, healthcare processes can be categorized based on how patient arrives, wait for service, obtain service, and then depart [2-4]. The healthcare processes also vary in complexity and extent, but they all deal with a set of both medical and non-medical activities and procedures that the patient must experience before getting the desired treatment. The servers in hospital queuing systems are the trained staff and equipment required for specific activities and procedures (Dushime-Agustin, 2015).

Queues are ubiquitous, particularly in the health care delivery system. At the same time, queues are undesirable because delay in receiving needed service can cause prolonged discomfort and economic loss when the patient is unable to work and possible worsening of their medical condition that can increase subsequent diagnosis and or treatment to the extent that death occurs while patients wait [5].

Adekunle Ajasin University, Akungba-Akoko (AAUA) is a Public University located in South-Western Nigeria, and the health-centre that services the health-care needs of the University community – staffs and students, is not an exception to the problem queues poses in health service delivery [6-8]. This work is therefore aimed at modelling the performance of the local health centre in terms of service delivery using Queuing theory, with a view to assessing the current state and suggesting measures to improve its performance.

2 Literature Review

Queuing theory deals with the study of queues which abound in practical situations and arise so long as arrival rate of any system is faster than the system can handle [9]. Queuing theory is applicable to any situation in general life ranging from cars arriving at filling station for fuel, customers arriving at a bank for various services, and in Health-care setting [10].

Queuing theory can be applied to the analysis of waiting lines in the health-care setting; queuing analysis can be used as a short-term measure or for facilities and resources planning. According to Adedayo [11] and Medhi (2003), queuing phenomenon comprises of the following basic characteristics: (1) arrival characteristics; (2) the queue or the physical line itself; (3) the number of servers or service channels; (4) queue discipline; (5) service mechanism; (6) The capacity of the system; (7) departure.

Brahimi and Worthington [12] and Goddard [13] modelled appointment systems and waiting list management in outpatient clinics. Tucker et al., (1999) used an M/M/1 model to assess operating room staffing needs for night shifts.

Peinus et al. (2000) used a non- preemptive priority queuing model to assess the average waiting times of emergent and non-emergent patients for computed tomography scans.

Nosek and Wilson [14] used queuing theory in pharmacy applications with particular attention to improving patient's satisfaction. Patients' satisfaction is improved by predicting and reducing waiting time and adjusting staffing plan. Gorunescu et al. [15] used a queuing model to help plan bed allocation in a department of geriatric medicine.

Vasanawala and Desser [16] used Poisson probabilities to predict the required number of reserved slots (on a weekly basis) for emergency radiology given that 95% of the requests are accommodated. Jonathan et al., (2009) characterized an optimal admission threshold policy using the control on the scheduled and expedited gateway for a new Markov Decision process model. In their work, they presented a practical policy base on insight from the analytical model that yield reduced emergency blockages, cancellations and off-units through simulation based on historical hospital data [17-20].

Vass and Szabo [21] used M/M/3 Model to characterize the patient flow in the emergency department. The study illustrated how data analysis and queuing can be used in decision making to obtain optimal service. Olorunsola et al. [22] used the M/M/C Model to model the flow of in-patient in hospital; determine the optimal bed count and its performance measure. Aziati et al. [23] used a descriptive analytical and simulation method to develop a suitable model. The collection of waiting time for this study was based on the arrival rate and service rate of patients at the outpatient counter. The data calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modelled and simulated using the ARENA software.

3 Methodology

3.1 Data collection

There were visits to various sections of the health-centre after approval and clearance from the administrator of the hospital. A stopwatch was used to calculate the number of minutes spent by each patient from the reception section where patients arrive and collect their hospital cards or register to the last section (the consulting room section). Data on the arrival time, waiting time and service time of each patient was collected on Weekdays (Mondays through Fridays) for three (3) weeks.

3.2 Formulation of the analytical model

The queuing model adopted for this work is the M/M/S model or Erlang C model which is the most commonly used queuing model [24]. The assumptions for this model are that there is a single queue with an unlimited waiting room that is going to be serviced by S identical servers. Patients arrive according to a Poisson process with a constant rate, and the service times have an exponential distribution [24]. The arrivals are serviced on a FCFS (First-come-first-served) basis [25].
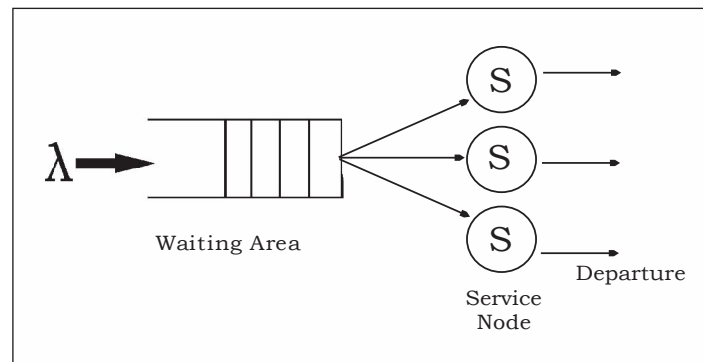

Fig. 1. M/M/S queuing model

3.2.1 The mean arrival rate

Let λ be the mean arrival rate and let n be the number of patients that entered the system. Also, let h be the number of observation hours. Then, the mean arrival rate is given by the formula,

$$\lambda = \frac{n}{h} \tag{3.1}$$

3.2.2 The mean service rate

At the history and the sections of the consulting room, patients were attended to one after the other. The service time for each patient was recorded when he or she is attended to.

Let $s_1, s_2, \dots s_n$ be the observed service time of patients. Let b be the start service time of patients and e be the finished service times of patients.

Then,

$$s_1 = e_{1-} b_1$$
$$s_2 = e_{2-} b_2$$
$$s_n = e_{n-} b_n$$

Hence, the mean service rate is given by;

$$\mu = \frac{1}{n} \sum_i^n s_i * \lambda_i \tag{3.2}$$

3.2.3 The mean waiting rate

For a system with a mean service rate μ, the average or mean service time is $1/\mu$.

Hence,

$$W = W_q + \frac{1}{\mu} \tag{3.3}$$

Where W_q is the average time in the waiting line and W is the mean waiting time in the system

$$P = \frac{\lambda}{s\mu} \tag{3.4}$$

where s is the number of servers and P gives an estimate of the utilization of the health centre i.e. the fraction of the system's service capacity that is being utilized in the average by arriving patients (λ).

L= Average number of patient's waiting for service $= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{p^2}{1-p^2}$ \tag{3.5}

Lq=Average time patients spent in the queue $= \frac{\lambda}{\mu-\lambda}..$ \tag{3.6}

The probability that the system shall be idle is given by:

$$P_0 = \frac{1}{\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu-\lambda}\right]} \tag{3.7}$$

The probability that there would be exactly n patients in the system:

$$P_n = \begin{cases} \frac{\left(\frac{\rho^n}{n!}\right)p_0}{} & n \le s \\ \frac{\rho^n}{(s!s^{n-s})p_0} & n > s; \ \rho = \frac{\lambda}{s\mu} \end{cases} \tag{3.8}$$

4. Results and Discussion

Calculations to predict the following performance measures analytically were carried using a Computer program written in Python language: the probability that there is no patient in system, the average number of patients in queue, the average number of patients in system, the average time a patient spends in waiting, the average time a patient spends in system and the probability that there is n number of patients in the health centre at a time (Po, n, L, Lq, Wq, W and P_n respectively) from the data collected.

All of these are necessary for effective scheduling of hospital resources and optimum planning.

Simulation experiments to obtain P_n assuming the values of n from 1 to 10 for each Week. Results for the concluding week is given in the Table below:

Table 1. Results of P_n for Week 3

	Mon	Tues	Wed	Thur	Fri
n	**Pn**	**Pn**	**Pn**	**Pn**	**Pn**
0	0.86687	0.88901	0.76639	0.76588	0.62676
1	0.14286	0.11765	0.26600	0.26667	0.46660
2	0.01020	0.00692	0.03538	0.03556	0.10886
3	0.00049	0.00027	0.00314	0.00316	0.01693
4	1.74E-05	7.98E-06	0.00021	0.00021	0.00198
5	4.96E-07	1.88E-07	1.11E-05	1.12E-05	0.000184307
6	1.18E-08	3.68E-09	4.92E-07	4.99E-07	1.43E-05
7	2.41E-10	6.19E-11	1.87E-08	1.90E-08	9.55E-07
8	4.30E-12	9.10E-13	6.22E-10	6.34E-10	5.57E-08
9	6.83E-14	1.19E-14	1.84E-11	1.88E-11	2.89E-09
10	9.76E-16	1.40E-16	4.89E-13	5.01E-13	1.35E-10
	$\rho = 0.04761$	$\rho = 0.03921$	$\rho = 0.086666$	$\rho = 0.08888$	$\rho = 0.15553$

Table 2. Performance metrics values at Week 3 (S =2)

Day	LQ	L	WQ	W
Monday	3475.58065	3478.96065	342.7594329	343.0927663
Tuesday	4654.447215	4657.897215	449.705045	450.0383783
Wednesday	1673621.678	1673625.345	152147.4253	152147.7586
Thursday	1083.995285	1087.795285	95.0873057	95.42063904
Friday	1169.684492	1173.501159	10.1558508	10.4891842

Further simulations were carried out to increase the number of servers from 2 to 3 to see the potential effect on performance delivery in the health centre. The results are given below:

Table 3. Performance values before Simulation to increase the number of servers

DAY	LQ	L	WQ	W
MON	3475.58065	3478.96065	342.7594329	343.0927663
TUES	4654.447215	4657.897215	449.705045	450.0383783
WED	1673621.678	1673625.345	152147.4253	152147.7586
THUR	1083.995285	1087.795285	95.0873057	95.42063904
FRI	1169.684492	1173.501159	10.1558508	10.4891842

Before Increasing the No of Servers (S = 2)

Table 4. Performance values after simulation to increase the number of servers

DAY	LQ	L	WQ	W
MON	233.3238395	236.7571728	22.65279995	22.98613329
TUES	837.7320872	841.2320872	79.78400831	80.11734164
WEDN	804.814011	808.5473443	71.85839383	72.19172717
THUR	1169.684492	1173.501159	56.1558508	60.4891842
FRID	80.23433757	83.38433757	8.490406093	8.823739426

After increasing the no. of servers (S=3)
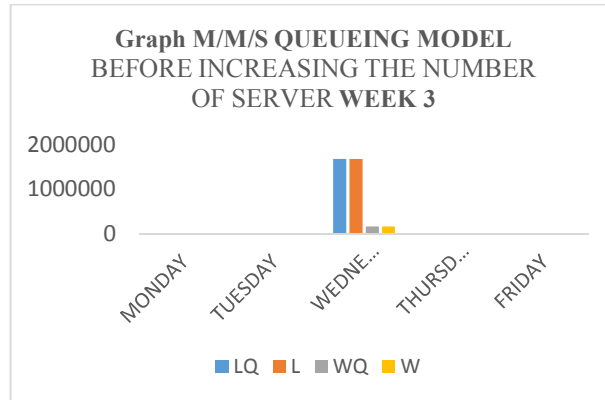
Fig. 2. Graph before simulation
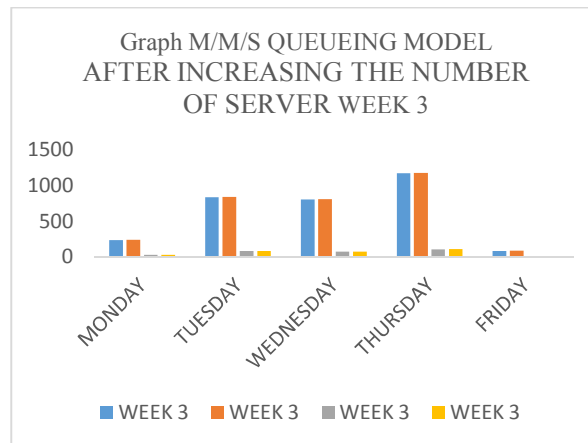
Fig. 3. Graph after simulation

4.1 Summary of performance measure

As we can see from the table and graph the results show that on Monday week3 the server would be busy 4.79% of the time and idle 86.68% of the time. Also, the average number of patients in the waiting queue before and after is 3475.58065 & 233.3238395 and the average number of patients waiting in the system before and after is 3478.96065 & 236.7571728, the average time a patient spends in the queue before and after is 342.7594329 &22.65279995 minutes and average time a patient spends in the system before and after is 343.0927663 & 22.98613329 minutes. For Tuesday the server would be busy 3.92% of the time and idle 88.909% of the time. Also, the average number of patients in the waiting queue before and after is

4654.447215 & 837.7320872 and the average number of patients waiting in the system before and after is 4657.897215 & 841.2320872, the average time a patient spends in the queue before and after is 449.705045 & 79.78400831 minutes and average a patient spends in the system before and after is 450.0383783 & 80.11734164minutes. For Wednesday the server would be busy 8.66% of the time and idle 76.639% of the time. Also, the average number of patients in the waiting queue before and after is 1673621.678 & 804.814011and the average number of patients waiting in the system before and after is 1673625.345 & 808.5473443, the average time a patient spends in the queue is 152147.4253 &71.85839383 minutes and the average time a patient spends in the system before and after is 152147.7586 & 152147.7586 minutes.

For Thursday the server would be busy 8.85% of the time and idle 76.588% of the time. Also, the average number of patients in the waiting queue before and after is 1083.995285 & 1169.684492 and the average number of patients waiting in the system before and after is 1087.795285 & 1173.501159, the average time a patient spends in the queue before and after is 95.0873057 & 10.1558508 minutes and average a patient spends in the system is 95.42063904 & 60.4891842 minutes. And lastly, for Friday the server would be busy 15.5% of the time and idle 62.67% of the time. Also, the average number of patients in the waiting queue before and after is 1169.684492 & 80.23433757and the average number of patients waiting in the system before and after is 1173.501159 & 83.38433757, the average time a patient spends in the queue before and after is 10.1558508 & 8.490406093 minutes and average a patient spends in the system before and after is 10.4891842 & 8.82373942 6 minutes.

Analysis of the system utilization factor of the servers at all the three weeks showed that among all the five number of days, the server of Friday week1 is the busiest of all with a utilization factor of 17.766%. It was also observed that Friday week 1 had the highest mean arrival of 5.33 patients/hour. This is understandable as Fridays make the close of work for the week and patients may likely develop health issues that needed to be attended to at the health centre owing to the accumulated stress of work across the days of the week.

Finally, it can be deduced that there was a considerably huge decrease in the waiting time and service time of patients when the server size is increased for each day of the Week. It is suggested that the number of physicians and nurses should be increased and a staffing plan should be developed in the health centre in order to manage efficient shifting of personnel.

5 Conclusion

Patients' satisfaction is very important to the hospital management because the patients are the people who sell a good image of the health centre to others which help to increase the revenue of the health centre. The objective of every hospital is to help reduce patients' waiting time, increase revenue and improve customer services and care.

The study looked at the queuing system at all the various sections of the AAUA health centre. It looked at patients' arrival rates, service rates and the utilization factor of the whole system with a view to observing areas that needed improvements and measures to be put in place to ensure sustained improvements and patients' satisfaction.

Competing Interests

Author has declared that no competing interests exist.

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